COMMUNICATION NETWORKS AND THE RISE

OF AN INFORMATION ELITE

CAN COMMUNICATION HELP THE INFORMATION RICH GET RICHER?

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"As a rule, the most successful people in life are those who have the best information" -- Disraeli

ABSTRACT: Freedom of access does not necessarily mean that information is freely shared. Thus universal access policies that provide only communication channels but do not also establish incentives for broader information sharing could widen the gap between information "haves" and "have-nots." This essay develops access and collaborative network arguments through a formal theoretical model based on the idea that an inability to explore increasingly rich opportunities for contact encourages individuals to focus their attention on their best opportunities. This can lead to the exclusion of alternative contacts. If the character of information makes dynamic interaction more valuable than one-way broadcast, then collaborative networks may form among those with the best opportunities. Based on factors such as the number of communication channels and the levels of learning and sharing that take place, the model rigorously explains how inter-agent dynamics can help the information "rich get richer" and also why "it's not just what you know but whom you know." It also suggests when networks of collaborators are stable over time. This theoretical framework serves to explain several stylized events and offers several useful insights into dynamic information sharing behaviors.

KEYWORDS: information networks, information economics, value added networks, national information infrastructure, sharing incentives, nonrival goods,

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I. Introduction

The promise of the World Wide Web is worldwide access -- linking people to information without barriers. In fact, national policy has long promoted "universal" access, arguing that "as a matter of fundamental fairness, this nation cannot accept a division of our people among telecommunications or information 'haves' and 'have-nots'" (NTIA, 1993). In industrialized communities, the gap in technology access continues to narrow. As new information is increasingly privatized (Branscombe, 1995), however, an interesting question emerges: does access to technology promote leveling or upheaval of the information landscape? While it is true that increases in communications and processing technology tend to generate more information (Pool, 1983), does it also follow that it will be more equitably distributed? This essay develops a model to simulate the behavior of collaborative information networks, and reveals conditions under which stratification becomes a possibility. The model also shows that equality of information access is a distinct possibility as well, but this result can depend on starting from relative equality. Under reasonable assumptions, the model predicts that greater connectivity can lead to greater stratification.

This framework distinguishes between access to facts and access to interactions that yield insights, and assumes that the best ideas come from people rather than from static data. If one's goal is to prevail in a complex legal proceeding, for example, access to a great law library is no substitute for access to a great lawyer. Analogous situations also emerge in business, politics, research, and scholarship; static information does not often replace expertise involving dynamic interaction. Communication technologies like the Internet represent more than access to information. They have the potential to create opportunities to interact with others for mutual benefit.

Knowledge exchange and interaction appear to be fundamental to tapping expertise. A treatise on knowledge asserts "... *we can know more than we can tell*.[1] ... We know a person's face, and can

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[1]Emphasis appears as in the original text.

recognize it among a thousand, indeed among a million. Yet we usually cannot tell how we recognize a face we know" (Polanyi, 1966, p. 4). Experts themselves cannot catalog their complete knowledge nor can they directly transfer inchoate ideas. One-way broadcast of useful information would require both a precise representation for un-formed concepts and a foreknowledge of every contingency. While interactive two-way communication cannot remove all the barriers to knowledge transfer, it creates opportunities for improved understanding, collaboration, and joint payoffs (Kofman & Ratliff, 1996). If interaction creates the greatest benefits, the question is not who will gain access to the best libraries, but who will gain access to the best authors, lawyers, physicists, medical researchers, etc. The availability of these expert resources is limited.

Time and competitive constraints on the availability of experts is compounded by bounded rationality, a limitation on human information processing capacity. As information technology increases access to information and simplifies the search for expertise, the explosion of choices intensifies the need to choose wisely. As Herb Simon suggests, "a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it" (Simon, 1973). The availability of fewer options for actual contact than for contact opportunities forces individuals to limit their choices.

What happens when unlimited access meets limited resources? If limits on choices apply to experts and non-experts alike, then under voluntary exchange private experts may choose with whom to share their knowledge. They may share with colleagues for free, with clients for a price, and with others only rarely, if at all. Those experts with the most valuable resources will typically expect the most compensation in return. If returns are then provided as reciprocal sharing, the information rich may leverage their existing knowledge to secure rich information.

Information technology (IT) can therefore have the unintended and ironic consequence that as more and more people become reachable electronically, competition for access can drive up the price of expertise. It can also promote commerce among widely distributed networks of experts who previously might have chosen to interact with local colleagues. Articles in today's business journals foreshadow the future of interaction, describing the interdisciplinary collaboration of elites across international boundaries (Young, 1995). In economics there also appears to be an increasing frequency of non-local coauthorships (Gaspar & Glaeser, 1996). If better information sources charge higher prices or have more prior commitments, then buyers need deeper pockets or better connections to secure their attention. At the top, an "information elite" might pool and exchange the best of the Internet.

With the proposed model of information and network interactions, this essay generates several interesting and policy-relevant observations. First, ubiquitous access can widen preexisting gaps in information resources if newly formed networks are stable over time. If networks start from relative equality there is less tendency toward stability and more toward homogenization. Second, different policies can encourage either partnering and sharing of information or isolationism and hoarding depending on how individuals are motivated. Incentives for seeking relative advantage discourage sharing while incentives for seeking absolute growth encourage it. Third, if information resources are interpreted as ideas, then the information output of networks substantially exceeds that of individuals even if the latter are blessed with equal or greater resources initially. The structure of contacts thus significantly alters the distribution of information over time.

These observations suggest behaviors that are also consistent with news reports and studies of group interactions, including stratification of newsgroups on the Internet (Chao, 1995), lack of information sharing within a firm (Orlikowski, 1992), winner-take-all markets brought about by increased access (Frank & Cook, 1995), benefits from access to quality universities (Daniel, Black & Jeffrey, 1995), and growth through information sharing in regional economies (Saxenian, 1994). The proposed information sharing mechanisms thus show a high degree of consistency with other models and a high level of explanatory power across a range of contexts. With the same model, it is possible to explore issues of peer groups, stability, network growth and topology, and momentum effects for persons excluded from a network.

The basic arguments are presented over the next six sections. Section §2 places the model and assumptions in the context of related literature while §3 develops the model and introduces parameters for each of the key constructs. A simple demonstration of the model in action follows in §4, leading to the development of a two period model regarding network formation and information exchange. Since temporal dynamics introduce novel agent behaviors and network instabilities, the model incorporates learning and depreciation in a continuous time version in §5. From the closed form solution to this problem, the essay moves to a few examples and comparative statics in §6. The final section comments on the model's scope and policies bearing on the general network and information framework in §7 before concluding with summary remarks.

II. Related Literature and Information Context

The information sharing model suggests that associations form among similarly ranked agents, calling to mind several studies on matching including the marriage model of Becker (Becker, 1973; Becker, 1974), the hospital intern model of Roth (1984), and the worker skill level model of Kremer (1993). In the present context, rising access facilitates matching among networks of information peers as agents gain the ability to communicate with one another.

In the network literature (Farrell & Saloner, 1985; Katz & Shapiro, 1985; Riggins, Kriebel & Mukhopadhyay, 1994), uncertainty creates friction, which discourages agents from switching networks even when it could be to their advantage. Under the proposed information sharing model, a modest amount of friction – in the form of time delay may also keep agents from switching networks despite potential advantages, and thus provides an alternative form of inertia. Network efficiency and stability principles are also explored in Jackson (1995). Under the proposed model, networks become increasingly stable as their resources become more stratified.

The model's information transfer mechanisms also highlight important features of winner-take-all markets where a handful of top performers can accrue a disproportionate share of the benefits as in Frank and Cook (1995). While broadcast markets allow all agents to gain access to the same information, the model demonstrates that only a handful of the best sources may be in a position to claim benefits.

Although information broadcast establishes equality among recipients, the pool of sources splits completely into high demand and no demand and inequality of opportunity prevails.

The sharing mechanism relies on both bounded rationality and a property unique to information: that it can be shared without consumption. Unlike tangible goods, such as cash or equipment, information is "nonrival" -- it is neither depleted nor divided when shared, and two or more agents can simultaneously consume a nonrival asset without physically displacing one another. Information can be reproduced almost without limit. A consultant or professor, for example, who gives advice to a colleague does not thereby reduce her knowledge through the interaction, in fact, her understanding might even improve. This property becomes important in explaining the dynamics of information exchange, as seen in Romer (1986; 1990), where nonrivalry leads to important macroeconomic explanations for increasing endogenous growth through knowledge spillover among firms. Information's no loss property means that multiple firms in an economy (or network) can benefit from the research performed by any one of them.

[2]Under the proposed model, however, agents may estimate the relative magnitude of one another's private information, but they may not know the actual content of that information without establishing contact with the owner.[3] A firm, for example, may know the publicly disclosed R&D expenditures of another firm but not know precisely how these funds are invested. Investing identical dollars does not guarantee identical nor even qualitatively equivalent knowledge and search costs may simply be too great relative to the value obtained. Thus one firm may require a joint venture or partnership with another firm in order to appropriate its know-how. In this model, agents know that more instrumental resources confer greater benefits and agents may be sorted on the basis of what they know.

This interpretation also draws strong support from the network organization, joint venture, and social network literature (Van Alstyne, 1997; Burt, 1993; Huber, 1991; Kogut, 1988; Powell & Brantley,

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[2] Described by Arrow as the problem of demand determination.

[3]It has been argued, for example, that scientific advancement would be much impaired if refereed journals were to accept results and not publish the manner of their achievement (Stephan, 1996). Emphasizing methods emphasizes the importance of an instrumental approach.

1993; Roberts, 1980) as firms and individuals are motivated to find partners for purposes of sharing information. Firms frequently "graft" knowledge bases by partnering because the time to derive the requisite knowledge through experience leaves the firm with a closed window of opportunity (Huber, 1991). Attempting to appropriate the know-how by imitation can also leave the firm with a shallow and imperfect understanding. Similarly, in the sociology literature, individuals can seek to leverage their opportunities on the basis of their own experience or "human capital," and on the knowledge of their associates or their "social capital" (Burt, 1993). Consultants, for example, frequently combine their own experience with that of their business networks to deliver on consulting projects.

In the organizational learning literature, information resources are improved or created by linking disparate sources, as when firms discover a shortage by juxtaposing inventory information with sales (Huber, 1991). Information technology (Jarvenpaa & Ives, 1994) and management literature also suggest how the benefits of pooling information can increase resource growth:

> ... knowledge is one of the few assets that grows most [when] shared. As one shares knowledge with colleagues, ... not only do [they] gain information ... they usually feed back questions, amplifications, and modifications, which ... add further value for the [sharer]. ... Since learning feeds knowledge back to the base, the next step (even at the same percentage increase) will spring from a higher base and be a larger absolute increment ... a principle embedded in ... experience curve or learning curve theories (Quinn, 1992, p 254).

As with genetic algorithms (Holland, 1992) and information genesis through the recombination of ideas (Weitzman, 1995), an agent's stock of information provides a useful and valuable asset for generating new ideas and information. Larger stocks provide a larger basis for combining ideas. Moreover, combining the diverse information of different agents can also foster growth. A large body of literature across a variety of disciplines articulates the advantages of an instrumental approach to information. Firms and individuals partner for information resources and generate new ideas on the basis of direct collaboration. Correspondingly, this also motivates the conditions for information sharing, which the proposed model seeks to describe.

To maximize their information resources, agents that are poor in information but rich in other capital could attempt to buy contacts – wealthy parents, for example, may buy their childrens' educations. A compelling advantage in one area often substitutes for an advantage in another. While the information sharing model admits the possibility of subsidized network entry through a resource exchange rate, the focus remains on access to information and on the closing of information gaps regardles of whether exchange rates provide opportunities for subsidy. In one sense, subsidy represents another form of access along with changes in technology, changes in willingness to communicate, and changes in sharing rates. The third example in a section seven offers two variations on this theme.

III. The Model

The basic model represents agents, information resources, sharing, networks, and channels. A link strength parameter also indexes the desire of one agent to communicate with another[4]. The continuous time model adds parameters for the genesis of new information and the obsolescence of old information.

Agents are designated i, j, k   I = {1, 2, ... |I|}. Each agent has access to a private endowment of valuable information resources e_i, the amount of private information available to agent i. Any public information can be added to or subtracted from these endowments without changing the relative gap e_i - e_j between any pair of agents. This focuses attention on private endowments which are assumed heterogeneous i.e. e_1 ? e_2 ? e_3 etc., implying that agents do not have access to the same information. When the time is introduced,$e_i(t)$ will refer to i's resources as a function of time. All agents have a finite number of channels, C, which they may use to access the endowments of other agents. Limiting the number of channels proxies physical, temporal, or cognitive capacity constraints. In addition, the reach or access parameter A describes technological access to other agents. If access is limited, agents may only reach near geographic neighbors chosen as a fraction of the population $\frac{|A|}{|I|}$, but if access is universal, agents may reach all other agents in I. Given capacity constraint C, however, increasing

[4]A glossary of symbols is included at the end of this article.

access implies that agents can actually communicate with at most max<A,C> of the reachable agents at any point in time.

Agents may share information with one another but they need not share everything at once, instead revealing their private information according to a sharing[5] parameter σ [0, 1]. Thus if agent i shares with j, then j gains access to σe_i but since i may share information without loss, e_i is not reduced. The no loss property captures the "nonrival" behavior of information.[6]

Agents in the model maximize a standard concave utility function u(e), with u'>0 and u''<0 by attempting to network with other agents having rich information endowments.[7] In private and voluntary exchange, one agent may also refuse contact with another offering too little reciprocal value. Let a link parameter L_{ji} [0, 1] index agent j's desire to communicate with agent i.[8] Then define the "network" of i, n(i), as the collection of agents with whom agent i has contact:

$$\mathbf{n(i) + \{i \gg j, j : L_{ij}L_{ji} > 0\}}$$

A link index of zero implies that j has vetoed all communication with i, thus both agents must be willing to communicate, $L_{ij} > 0$ and $L_{ji} > 0$, for a channel to provide either agent access to the other's information. A completely open channel requires that $L_{ij}=L_{ji}=1$. For agent i, communication with everyone implies that $\forall$ i and j, links $L_{ij} = 1$, that i has at least (|I| - 1) channels, and that n(i) = I = {1, 2, ... |I|}. In general, however, for C < I the members of i's network form only a subset of all agents, those who share a mutual willingness to communicate with I.

Using these concepts, define an information "eliteness" relation to mean agents who have greater access to resources through their own private endowment *and* through those resources separately held by their associates. Formally, i is at least as elite as j if and only if i's total resource access is at least as great:

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[5]Alternatively, this can be viewed as measure of "secrecy" equal to (1-σ).
[6]This framework still permits the modeling of relative changes as when i loses in strategic advantage relative to j.
[7]Concavity is not necessary for the principal results as a convex utility function strengthens the attrativeness of agents with more resources.
[8]Agents are assumed to have access to their own resources thus $L_{ii} = 1$ throughout.

$$\mathbf{i \gtrsim j + R(i, t, I) = R(j, t, I) \text{ where } R(i,t,I) + e_i(t) + s \;{}^{I}_{k?i} L_{ik}\, L_{ki}\, e_k(t)}$$

R(i, t, I) thus designates the total resources of i at time t, inclusive of private and shared resources. Since the argument I carries across all agents, the third argument will be suppressed. The constituents of R(i, t) have notable parallels with economic and sociological concepts, namely human capital and social capital (Burt, 1993). Human capital, in this case $e_i(t)$, refers to individual resources, for example knowledge, education, and experience, whereas social capital, here $\sigma \sum_{k°i} L_{ik}L_{ki}e_k(t)$, refers to the value an agent captures through his or her connections to other people, for example, a contact's knowledge of emerging problems and opportunities. For clarity, it is often convenient to write $e_{-i}(t)$ as shorthand for the total endowment of all agents in i's network of contacts not including i which would otherwise be written less succinctly as $\sum_{j \in n(i),\ j°i} e_j(t)$. Within this framework, the general model assumes:

(1) (*Bounded Rationality*) – Agents cannot communicate with everyone at once; channels per agent C are finite and fewer in number than the population, $C < I - 1$.

(2) (*Initial Inequality*) – Endowments are randomly distributed on $e_i(0)$ (0, K] where K is in $—^+$ giving positive support.

(3) (*Nonrivalry*) – Information is not lost in sharing; if j shares fraction σ [0, 1] with i then i's resources increase by $e_i(t+\Delta t) = e_i(t) + \sigma e_j(t)$ but, in the absence of other effects, $e_j(t+\Delta t) = e_j(t)$.

(4) (*Learning and Depreciation*) – Agents may generate new ideas and information on the basis of information to which they have access, i.e. there exists production technology $\Phi(i, t, I) = \lambda[e_i(t) + \sigma \;{}^{I}_{k?i} L_{ik}\, L_{ki}\, e_k(t)]$ where λ $—^+$. Agents may also forget information with time or it may become obsolete at a depreciation rate δ [0, 1].

The first assumption guarantees that an agent cannot contact every other agent in any single period; thus it represents the necessary limitations on attention, physical capacity, or time. If agents cannot communicate with everyone at once, this forces them to make choices regarding with whome to communicate. It describes the case where everyone has the same technology and is fairly conservative since allowing richer agents more channels would strengthen a conclusion that the rich get richer, an advantage which is unnecessary in proving the main result. Uniformity of channels also more accurately reflects a broader interpretation of "universal access" in which everyone is granted access to the same communications technology. The second assumption simply asserts that agents do not start out with level resources. Agents may have similar channels, but they have heterogeneous information. The third assumption implies that agents can share information without losing it themselves. This assumption characterizes low cost duplication and still allows the model to acount for the possibility that j loses in strategic advantage relative to i -- a phenomenon modeled explicitly in Proposition Three. Assuming that information is nonrival also implies that once agents have shared information, they cannot effectively pull it back from recipients. With respect to the third assumption, it will often be convenient to refer to σ as a sharing parameter in contrast to the learning parameter λ .

The fourth assumption furnishes a mechanism for information to enter and exit the system. More precisely, agents may improve their resources in proportion to the resources to which they have or can gain access. Thus production inputs involve both private endowments $e_i(t)$ and shared endowments $\sigma e_{-i}(t)$ or both human and social capital in the terms of sociology (Burt, 1993). Since $\Phi'(e_i(t)) > 0$, securing partners with more resources creates an opportunity for an agent to learn more. It also marks more expert agents as more attractive partners. The depreciation term δ allows the model to account for the possible obsolescence of static information. Although this formulation implements a specific functional form for ease of modeling, Proposition 5 shows that other quite general forms do not necessarily alter the main results.

For the key propositions, a simple two period analysis suffices to make the point that freedom of contact does not mean information is freely shared. These arguments are based on only the first three assumptions. For the more involved stability propositions, the model functions in continuous time after taking account of assumption four.

IV. An Illustration

For illustrative purposes, we now consider the choices and network behaviors in the specific case of 100 agents and 4 communication channels per agent starting in period zero with no network connections. Without loss of generality, we number the agents, sorting them on the basis of their endowments. The richest endowment corresponds to rank one and the smallest to rank 100, thus agent i has more information than j for i closer to one. All agents prefer to communicate with the first agent since $u(e_1) > u(e_2) \ldots > u(e_{100})$. Since agent 1 only has C channels and because he has veto power, he chooses only to communicate with agents 2 through C+1 = 5 whose shared resources offer the greatest return. Agent 2, having C-1 channels remaining, chooses the next best agents 3 through 5. With one fewer channels still, agent 3 repeats this process, choosing agents 4 and 5. The first network therefore contains only agents 1 through 5. As the first five agents are now unavailable, agent 6 becomes the most attractive partner. Filling all channels for the next four most attractive candidates constitutes the second network inclusive of agents 6 through 10 or C+2 to 2C + 2. This process repeats for agents 2C + 3 to 3C + 3. Networks one through twenty[9] emerge:

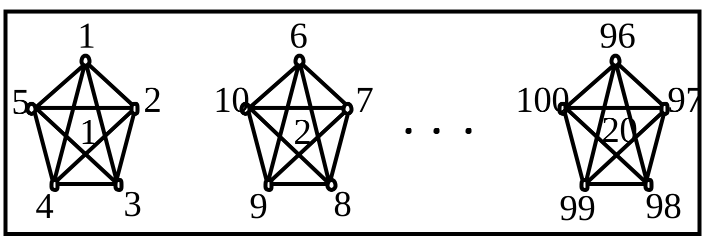

Figure 1 -- Given limited channels, agents form exclusive networks.

Prior to network formation, the fifth agent is as elite as afterwards -- no one has changed rank on the basis of newly shared resources. The gap between the fifth and sixth agents, however, has increased from e_5 - e_6 to

$$e_5 - e_6 + \sigma[(e_1 + e_2 + e_3 + e_4) - (e_7 + e_8 + e_9 + e_{10})]$$

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[9] If I/(C+1) is not an integer, the final network is incomplete.

since the fifth agent has contacted richer endowments. Although the gap between agents five and six increases with network formation, note that the gap between agents four and five, as given by $(e_4 - e_5)(1 - \sigma)$, diminishes for any σ above complete secrecy. Other members of their joint network are the same. Also, even though the instrumental framework permits agents to be sorted, agents need not agree completely on the precise ordering. Agents need only agree on the relative tiers for identical networks to form and, to a first approximation, even mild disagreement on tiers preserves the basic result of convergence within networks and separation among them.

The example illustrates intuitive results which we now make explicit. Each agent, acting in his own self interest, pursues a strategy of maximizing his resource contacts. Agent i seeks to access endowments e_j by selecting high values for L_{ij} according to[10]:

$$\max_{L_{ij}} u(\sum_{j \neq i}^{I} L_{ij} L_{ji} s e_j) \text{ such that } |n(i)| \leq C+1 \text{ and } L_{ij}, L_{ji} \in [0,1]$$

The share of other agents' endowments that i can access is given by $\sigma L_{ji} e_j$ for any partner j. Since agents participate in their own network, here $|n(i)| = C+1$ prevents i from using more than C available channels. In solving for the strategies of network agents, the following formulas will prove useful in making assertions about agent and network rank. The function f(i) designates the rank of the network joined by agent i where $f(i) < f(j)$ implies $R(i, t) > R(j, t)$ since i participates in a higher rank network.

$$\boxed{f(i) \equiv \left\lfloor \frac{i-1}{C+1} \right\rfloor + 1}$$

This returns the integer portion of the quotient. Conversely, we can define the inverse function g(n) to give the rank of the first agent[11] from network n. The last agent in a network can then be determined by computing g(n) + C. This function is defined as:

$$\boxed{g(n) \equiv (C+1)(n-1)+1}$$

[10]An alternative formulation of the objective function is described in Proposition 3.
[11]This is not a true inverse in the sense that the mapping is not one-to-one.

Using the functions to claim, for example, that the first agent of a network has greater resource access than the last we write R(g(n), t) > R(g(n)+C, t). Given any arbitrary agent, the most elite member of his network may be written as g(f(i)) or equivalently as[12] min[n(i)]. Thus two agents i, j where i?j have the same network rank if and only if f(i) = f(j). When this condition holds, some common information will be shared between them since agents i and j clearly share this information, σe_i and σe_j, with themselves. This is true of all the members of n(i); thus there exists a common resource pool $z_{n(i)}$ where $\forall j$ n(i), j receives $z_{n(i)} = \sum_{k \in n(i)} \sigma e_k$. With appropriate subscripts for the first through last agents of i's network this would be represented as $\sigma(e_{g \circ f(i)} + e_{g \circ f(i)+1} + \ldots e_{g \circ f(i)+C})$ which is simplified to $\sigma e_{n(i)}$. When it is obvious i belongs to network n, the additional notation is suppressed and pooled resources are written simply as $z_n = \sigma(e_1 + e_2 + \ldots e_{C+1})$

V. A Two Period Model

In the basic mode, agents start with no connections in period zero, corresponding to no technological access. They choose partners, in any order, then share information in period two. Agents are free to associate voluntarily in the sense that they may choose the best partnership option available and potential partners have veto power. Intuitively, then, an agent i chooses the C best options from the menu of values in $\sigma e_j L_{ji}$, with j=1, 2, ... i-1, i+1, ...|I|. This choice gives rise to:

Proposition 1.A (*Principle of Growing Network Inequality*): Utility maximization in voluntary networks increases the resources of the information elite i.e. $\forall$ i < j in period 1, R(i, 1) - R(j, 1) = R(i, 0) - R(j, 0). The following strategy is a Nash equilibrium:

$$\text{strategy(i)} = \begin{Bmatrix} L_{ij} = 1 & f(i) \geq f(j), i \neq j \\ L_{ij} = 0 & f(i) < f(j) \end{Bmatrix}$$

implying that agents prefer to communicate with other agents having equal or greater endowments than those in network f(i).

Proof: The proofs are provided in Appendix A.

[12]Although max[n(i)] might seem more intuitive, the fact that |I|/(C+1) is not always an integer makes equations using a reverse ordering |I|. |I|-1, ... 2, 1 unwieldy. It is preferable to think of the top agent as 1, the second best as 2, etc.

Proposition 1.B (*Principle of Growing Equality Within Networks*): Preexisting differences in resource access among agents in the same network decrease. That is for f(i) = f(j), R(i, 1) - R(j, 1) = R(i, 0) - R(j, 0).

Intuitively, agents aspire to their best partnerships and the best partners choose one another. The problem characterized by Proposition 1 make popular news. A handful of electronic groups have become privatized with membership and participation limited "to people with valuable knowledge" (Chao, 1995, p. B1). Similarly, the newsgroup sci.physics

... is home to a mishmash of physicists, well-meaning amateurs, and flat out cranks. The result is sometimes interestingly eclectic, but the process [of interaction] often leads to acrimony, as the cranks drown the newsgroup in tirades about time travel, experts try to stem errors and misconception -- eventually admitting defeat and retreating to smaller, more specialized newsgroups or invited e-mail lists. (Steinberg, 1994, p. 27)

The effects of Propositions 1A and 1B have also been observed in business practice. In a five year study of US, European, and Japanese joint ventures, Hamel, Doz and Prahalad found that: "A strategic alliance can strengthen both companies against outsiders even as it weakens one partner vis-a-vis another" (1989, p.133). By sharing information, companies improve their total resources yet reduce the gap between them.Interpreting universal access as broadcast, information technology can potentially level access to information by offering everyone access to identical information. Globally broadcasting a private resource reduces the gap between haves and have-nots whenever one group of agents previously enjoyed access denied to others. This is likely to represent the most significant gains from universal access, although it can also depend on motivating private owners to share their sources of private advantage.

Publicly broadcasting *new* resources, however, tends to raise the resource level for everyone rather than closing the gap between top and bottom. The ratio improves but the gap remains unaffected. In addition, two other factors can mitigate the benefits of broadcast: it can actually concentrate the opportunities for partnering among the elite and not all information can be broadcast.

Broadcast lifts the distribution constraint on the best sources of information so that they might possibly serve the entire population. Although universal broadcast grants everyone access to the best resources, it subtly and simultaneously reduces the opportunity of all but the elite to trade on the basis of these resources. This leads to the following proposition.

Proposition 2 (*Winners-Take-All Effect of Increased Access*): Given a capacity constraint C << |I|, increasing access raises the probability that more elite agents become more attractive sources. It can also lower the probability that less elite agents remain attractive sources. More precisely, let B represent the event that j is an attractive source indicating that other agents prefer to contact j. Then, p(B | j = C) -> 1 and p(B | j > C) -> 0 as A -> I.

The proof uses a combinatoric argument to show that as access increases, the likelihood of the first C agents appearing in any other agent's top C choices will approach one, while for any agent higher than C the likelihood approaches zero. Plotting the equations that define these relationships also gives a more direct impression of what happens to attractiveness as access improves. Figure 2 plots the case of |I| = 25 and C = 5 and shows the probability that an agent appears in the first C choices of any other agent.

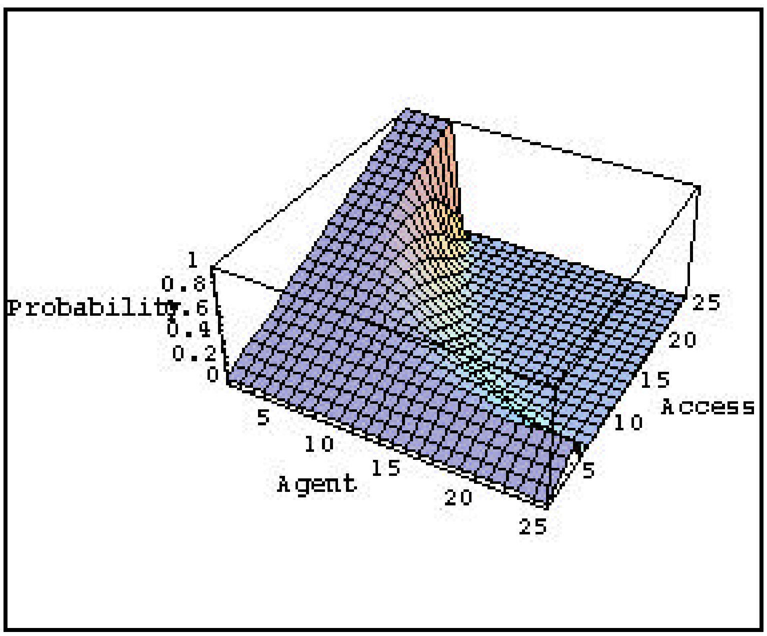

Figure 2 -- Increasing access benefits everyone until the capacity constraint binds; then the top C agents benefit most.

The "winners-take-all" effect of increasing access becomes apparent when the last row, with access at 25, shows that no agent ranked higher than C is an attractive source relative to the first C agents. All probabilities in this row are either 0 or 1. The only way to mitigate this effect is to also boost C as A->I; that is to lift the capacity or bounded rationality constraint that prevents agents from communicating with the entire population.

We can use a Herfindahl index,[13] to illustrate the rising concentration of opportunity among elites with changes in access. Here, we ask what is the concentration of suppliers of information? At any given instant, each agent seeks C suppliers and there are |I| agents, thus there are C⊇|I| units of demand for contact. Prior to universal access, all agents are interested in their neighbors' resources, thus each agent has a demand of roughly C. The concentration index of opportunities for contact is therefore H=$\sum_1^I (\frac{C}{C \cdot |I|})^2 = \frac{|I|}{|I|^2} = \frac{1}{|I|}$. Following universal acccess, broadcast permits each of the top C agents to serve the entire population I. No one else has any market share. The concentration of contact opportunities rises to H = $\sum_1^C (\frac{|I|}{C \cdot |I|})^2 = \frac{C}{C^2} = \frac{1}{C}$. By assumption, bounded rationality forces C << |I|, so that opportunities are vastly more concentrated among the elite ($\frac{1}{C} >> \frac{1}{|I|}$) under universal access.

Figure 2 shows that the attractiveness of reaching any agent at all increases with improving access until the capacity constraint limits the number of simultaneous contacts. As the capacity constraint binds, starting at Access = Capacity = 5, the least well off are the first to be adversely affected. Then, as access continues to improve, the information "middle class" eventually become affected by the capacity constraint. In the end, only the most elite agents face positive prospects. To illustrate this phenomenon, Frank and Cook (1995) cite a Vonnegut novel in which a local musician stops playing as his customers gain access to a broader selection of musicians. "A moderately gifted person who would have been a community treasure a thousand years ago has to give up, to go into some other line of work, since modern communication has put him in daily competition with nothing but the world's champions" (p. 1).

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[13]This measures market share concentration among firms within an industry. Let α_i be the market share of firm i and measure H as $\sum_i a_i^2$. Then H (0,1], it decreases in the number of firms that have positive market share, it approaches its minimimum when each firm has equal market share, and it reaches it maximum when one firm has the entire market [Tirole, 1990 #321].

Broadcast permits everyone access to the same information. Everyone may watch the game, but only the elite have a chance to play for a living.

The entertainment analogy highlights the second mitigating factor that not all information can be broadcast. As Polanyi's epistemological observations suggest, access to tacit expertise is feasible only through direct mediation on the part of the expert. Then, assuming it were possible to render all information explicit and accessible, an expert might still be more efficient. With expert counsel, for example, a person could sort through 2^{20} possible results in only 20 yes/no questions (i.e. $< 2^5$). Unstructured search would require an average of $(1/2)2^{20}$ queries to locate the same result or 2^{19}, that is 2^{14} times *more* search effort. The best attorney can not serve at court proceedings for each member of a population via one-way broadcast. Interaction and two-way communication create opportunities for improved understanding, collaboration, and joint payoffs (Kofman & Ratliff, 1996).

The results of Propositions One and Two rest on the assumption that agents maximize their absolute resources. Shifting to the maximization of relative resources allows us to explore what happens when agents seek strategic advantage -- that is, to improve their position vis-a-vis other agents. We model this as an agent deriving utility from the gap in resources $e_i - e_j$. With $u' > 0$, agents prefer to grow any advantage $e_i > e_j$ and shrink any disadvantage $e_i < e_j$. Thus, pursuing strategic advantage leads to

Proposition 3 (*Information Climber Effect*): If agents maximize relative assets in lieu of absolute assets, information sharing halts. That is, the following strategy[14] becomes a Nash equilibrium:

$$\text{strategy (i)} = \begin{Bmatrix} L_{ij} = 1 & j < i \\ L_{ij} = 0 & j > i \end{Bmatrix}$$

Like a "social climber," an agent who employs a relative value strategy prefers to partner only with agents whom they perceive to have a higher rank. The intuition underlying the result that agents stop sharing is that agents at the top have a disincentive to share since it reduces their advantage at time t + 1 from the advantage they enjoyed at time t. As this behavior cascades down from more elite agents, a

[14]Given that L_{ij} only takes values of zero or one in both absolute and relative strategies, it is suppressed in further discussions.

system which rewards the seeking of relative advantage prevents information sharing. A study of groupware introduced into a consulting firm observed this phenomenon (Orlikowski, 1992). In a competitive up-or-out atmosphere, consultants below the level of principal would vie for a limited number of promotions partly on the basis of individual competency. Revealing a unique source of expertise risks shrinking any relative advantage over less qualified candidates or growing the advantage of more qualified candidates if the beneficiary does not respond with at least as much valuable information.

Ironically, the same firm provides evidence of both absolute and relative objective functions in different contexts. At the firm's highest level, principals enjoy permanent tenure and focus more on absolute rather than relative maximization. Among principals, collegiality and information sharing prevail. Different incentives and behavior indicate the existence of their separate agendas. Consultants "feel little incentive to share their ideas for fear that they may lose status, power, and distinctive competence. Principals, on the other hand, do not share this fear and are more focused on the interests of ... the firm than on their individual careers (Orlikowski, 1992, p. 367)." Given that sharing halts with a relative objective function, subsequent propositions return to the assumption of absolute maximization.

VI. A Continuous Time Model

A continuous time model introduces dynamic behaviors for agents seeking to maximize their resources. Under certain conditions, for example, agents may gain all of their partners' private information, then prefer instead to form tree structures[15] or to alternate among different partnerships. If, for example, there were no learning and agents were to share everything, then partner hopping becomes optimal each period since all possible sharing has taken place. On the other hand, in an enviornment of no sharing but high learning, agents might form no connections at all. Finally, in an environment of high depreciation, stocks of information become obsolete leading to stable networks among partners with high learning rates. This section examines factors leading to network growth and stability.

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[15]We thank an anonymous reviewer for an example in which each agent prefers to connect to agents that are not yet connected.

To increase dynamic fidelity, the continuous time model incorporates learning parameter λ, allowing new information to enter the system, and depreciation parameter δ, allowing obsolete information to exit the system. Total resources might therefore grow as a function of private learning $\lambda e_i(t)$, sharing from partners $\sigma e_{-i}(t)$, learning from shared information $\lambda\sigma e_{-i}(t)$, and from depreciation $-\delta e_i(t)$. A more complete equation for resource growth might therefore be

$$e_i'(t) = l\, e_i(t) - d e_i(t) + s\, e_{-i}(t) + l\, s\, e_{-i}(t) - d s\, e_{-i}(t)$$

This description, however, permits resources to be *re*shared. A partner who shares $\sigma e_j(t)$ in period t receives back shares $\sigma^2 e_j(t)$ in period t+1. A pooled resource term $z_n(t) = \sigma[e_1(t)+ e_2(t)+ \ldots e_{C+1}(t)] = \sigma e_n(t)$ can be introduced to eliminate this double counting. The more accurate formulation shifts partner-shared resources $\sigma e_{-i}(t)$ to $z_n(t)$ as well as shifting $\sigma e_i(t)$ to $z_n(t)$. Agents then learn from $z_n(t)$, but do not reshare the same resources. Accounting for pooling generates alternate continuous time functions.

$$e_i'[t] = (l - s - d)e_i[t] + l\, z_n[t]$$
$$z_n'[t] = s\left[e_1[t] + e_2[t] + \ldots e_{c+1}[t]\right] - d z_n[t]$$

Accounting for the transfer of private resources $\sigma e_i(t)$ from the first equation to the second does not alter the nonrival assumption. Agent i retains full access to resources $R(i, t) = e_i(t) + z_n(t)$. Individually, agents have less private information but gain access to all resources shared into the pool. The pooled resource term also preserves the interpretation of $e_i(t)$ as a strictly private resource. For accounting purposes, this merely eliminates resharing of the same resource. The preceding equations then define a system of ordinary differential equations.

Lemma 4 -- The system of ordinary differential equations have a closed form solution

$$e_i(t) = e_i(0)\,E^{t(l-s-d)} + \frac{e_n(0)}{c+1}\left[-E^{t(l-s-d)} + \frac{1}{2r}\left((l-s+r)E^{\frac{t}{2}(l-s-2d+r)} - (l-s-r)E^{\frac{t}{2}(l-s-2d-r)}\right)\right]$$

for private resources and

$$z_n(t) = \frac{s\, e_n(0)}{r}\left[E^{\frac{t}{2}(l - s - 2d + r)} - E^{\frac{t}{2}(l - s - 2d - r)}\right]$$

for pooled resources, where r = $\sqrt{(l + s)^2 + 4c\, l\, s}$ simplifies the terms under a radical.[16] The pooled resources $z_n(t)$ depend on σ > 0 yet they strongly influence the growth in private resources $e_i(t)$ as represented by the final two exponential terms. The larger of these two exponents, (λ - σ - δ + r)/2, tends to be larger than the exponent on the initial private term, (λ - σ - δ). The former approximates the group contribution to private growth while the latter approximates the private gains achieved by individuals. In the absence of sharing, individual growth would be limited to $e_i(0)\,E^{l-d}$. This individual contribution to one's own private growth can be quite small since the individual terms cancel in the sum of the C+1 members' private resources:

$$e_n(t) = \frac{e_n(0)}{(c+1)2r}\left((l - s + r)E^{\frac{t}{2}(l - s - 2d + r)} - (l - s - r)E^{\frac{t}{2}(l - s - 2d - r)}\right)$$

This solution exhibits several interesting properties. Assuming no depreciation, for example, a four agent network with initial endowments {$e_1(0)$, $e_2(0)$, $e_3(0)$, $e_4(0)$}, λ = 0, and σ = 1 has resources that behave according to

$$e_i(t) = e_i(0)\,E^{-t} \text{ and } z(t) = \left[e_1(0) + e_2(0) + e_3(0) + e_4(0)\right]\left[1 - E^{-t}\right].$$

This implies that all private resources enter a common pool but the network's total resources never increase. Similarly, if λ = 1, and σ = 0, then the private and pooled resources behave according to

$$e_i(t) = e_i(0)\,E^{t} \text{ and } z(t) = 0$$

implying that agents gain new resources entirely through independent learning. The first example leads to a question of when networks remain stable. This is given by a test for stability.

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[16]From the fact that λ. σ, and c = 0, it follows that r = 0. The relative magnitudes also ensure that (λ - σ + r) = 0 and (λ - σ - r) = 0, inequalities that will prove important for subsequent derivations.

Proposition 4 (*Convergence and Stability Conditions*) -- For any two agents i and j, $e_i(t)/e_j(t) \to e_{n(i)}(0)/e_{n(j)}(0)$ subject to the condition

$$E^{tr} < \frac{[(e_n - e_{n+1})(l - s - r) - 2s(c+1)e_{n+1}]}{[(e_n - e_{n+1})(l - s + r) - 2s(c+1)e_{n+1}]}.$$

If resources grow and networks are stable, it also follows that if f(i) = f(j), then $e_i(t)/e_j(t) \to 1$ whereas for f(i) < f(j) then $e_i(t)/e_j(t) \to K > 1$ and $e_i(t) - e_j(t) \to 8$.

Networks passing below the threshold will reconfigure.[17] Each parameter in the test is positive thus all terms in parentheses are positive with the exception of ($\lambda - \sigma - r$) which is always non-positive. Therefore, the numerator must be negative. Since E^{tr} must be positive, the test 's feasibility hinges on also having a negative denominator or $(e_n - e_{n+1})(\lambda - \sigma + r) < 2\sigma(c+1)e_{n+1}$. Dividing through by a negative denominator also switches the sense of the inequality so that E^{tr} must grow larger than the cutoff rather than smaller. Time never appears on the RHS so the test must either fail a negative threshold immediately or eventually pass any positive threshold. If the test fails, networks are permanently stable. If the test will pass, the optimal time for reconfiguring the network can be found by taking logs. The following table lists stability implications of Proposition 4.

Condition	**Time to Instability**	**Description**
$e_n \to e_{n+1}$	0	In a population of peers, agent networks do not stratify.
$e_n >> e_{n+1}$	Never	If initial conditions are widely divergent, networks permanently stratify and endowments separate further with time.
c -> 8	0	Increasing the number of connections makes it more likely that networks will reconfigure with new agents.
$\sigma \to 0$	Never	In lower sharing environments, networks will be more stable.
$\lambda \to 0$	$\frac{e_n + ce_{n+1}}{e_{n+1} + ce_{n+1}}$	In lower learning environments, seeking new sources for new information eventually becomes optimal.
$\lambda \to 8$	Never	High learning rates make networks permanently stable.

[17]The condition is a strong test in the sense that networks below the threshold will always reconfigure while networks near the threshold will usually (but not always) not reconfigure.

Proposition 4 combines the properties of Propositions 1.A and 1.B with an additional test for whether partnerships remain stable. If they are stable, the gap in intra-network resources shrinks while the gap in inter-network resources grows. The instability condition assumes no cost to dissolving the existing network (extrication costs, cancellation fees, contracting penalties, etc.) and no cost to brokering a new network (search costs, negotiation costs, startup fees, etc.). Adding either type of cost increases the stability of prevailing connections.

One unusual property is that δ does not appear in the test. Technically, δ only appears along the main diagonal of the square matrix of resource change rates, thus it does not influence the eigenvectors of the solution. More intuitively, δ affects all resource stocks equally. The key factor, resource growth, depends on the interplay of learning λ, sharing σ, and network size C, which are captured in the test for stability. Of these, learning is the most important and low learning rates do, in fact, lead networks to dissolve as the table suggests. Under one interpretation, higher δ may be seen as inducing lower λ -- an effect that is already captured.

Although the formulation uses a linear production or learning function, λ (Assumption 4), to simplify the model, the key results do not depend on this particular formulation. A concave but positive growth production technology will yield similar results in the sense that resources will increase at an increasing rate. As in Romer (1990), the driving force is the nonrival property of information goods. This leads to the following observation:

Proposition 5 -- (*Principle of Convex Growth*) Let $\Phi(x)$ be any concave production technology such that $\Phi' > 0$ and $\Phi'' < 0$ i.e. output increases with input but exhibits decreasing returns to scale. Assume also that $\Phi(x) > 0$ so that Φ does not produce negative output. Let $\Psi(t \mid e(0))$ be the information state describing the available information when Φ is the production technology. Then Ψ can be a convex function of time with both $\Psi' > 0$ and $\Psi'' > 0$. Since concave technology is the most difficult such case, *a fortiori*, any positive growth production technology Φ also gives to a convex information state Ψ regardless of its convexity properties.

This implies that even with a decreasing returns to scale production technology, the available information exhibits increasing returns to scale over time. Of course, if the production technology exhibited increasing returns to scale this effect might be larger still. Thus the implications of Proposition 4 regarding network separation do not depend on the particular formulation of the model we have chosen to adopt. Our key result, that of increased stratification, is robust to alternative formulations meeting the criteria put forth in Proposition 5. In fact, if one tracks information over time, crude measures of growth patterns match the convexity properties suggested above rather well. Growth in communications across different media exhibits this pattern (Pool, 1983). A logarithmic plot of volumes at major research libraries (Cummings, Witte, Bowen, Lazarus & Eckman, 1992) represents another example. The growth rate has been relatively constant despite financial limitations on physical acquisitions.

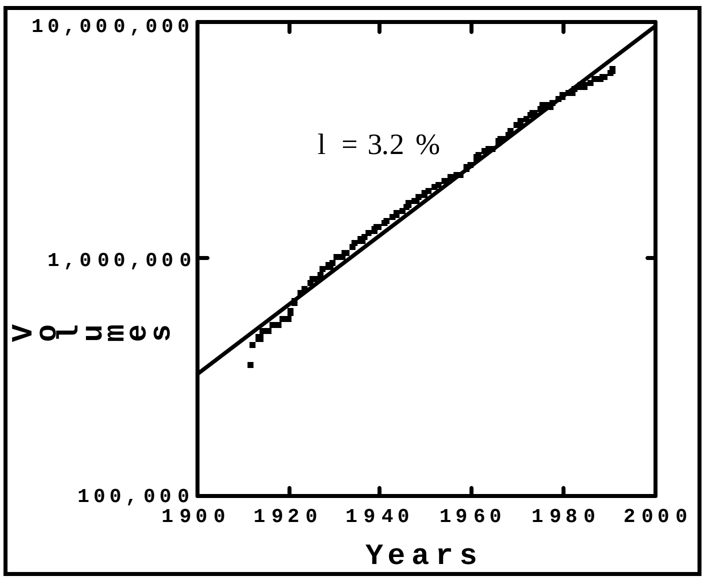

Figure 3 -- Growth in volumes at private research libraries has exhibited convex growth for most of the 20th century.

A further implication of this result is that borrowing to enrich a poor endowment becomes more difficult when information resources are important. Under normal circumstances, a concave production technology allows someone starting lower on the output curve to produce at a marginally higher rate even if their total output is smaller. The same increment to scale creates more value lower on the curve, thus investment borrowing ought to be relatively easier since lenders might also earn more. These positions are reversed, however, if resources exhibit increasing returns to scale -- agents higher on the curve face better investment prospects. This makes an initial disadvantage more difficult to overcome, further concentrating resources at the top.

VII. Examples & Comparative Statics

Changes in initial endowments, learning rates, and sharing rates can lead to large differences in network membership and growth over time. The following examples present the effects of changes in the values of the private and pooled resource formulas.

A.1. Example: Initial Conditions Influence Stratification

Under expanded access, widely heterogeneous initial conditions can lead to growing stratification. Let there be universal access in a population of I = {40, 30, 20, 10, 4, 3, 2, 1}, with C = 3, and $\lambda = \sigma = \delta = 1/2$. Then $e_{n=1}(0) = 100$ and $e_{n=2}(0) = 10$ with growth characterized by

$$e_i(t) = e_i(0)E^{-t/2} + \frac{e_n(0)}{4}\left[-E^{-t/2} + \frac{1}{2}\left(E^{t/2} - E^{-3t/2}\right)\right]$$

$$z_n(t) = \frac{e_n(0)}{4}\left(E^{t/2} - E^{-3t/2}\right)$$

The highest growth private term is $\frac{e_n(0)}{8}E^{t/2}$ and the highest growth pooled term is $\frac{e_n(0)}{4}E^{t/2}$. Members of network one therefore have private growth on the order of $\frac{100}{8}E^{t/2} = 12.5E^{t/2}$ whereas members of network two have combined private and pooled growth on the order of $10(\frac{1}{8} + \frac{1}{4})E^{t/2} = 3.75E^{t/2}$. Accounting for both types of resource, members of network one do not find it optimal to forge links to members of network two in subsequent periods.

Prior to universal access, networks might still include members with heterogeneous initial endowments if endowments were dispersed and networks were physically separated. In this example, the expected endowments of two arbitraily chosen and separated networks would both be $e_n(0) = 55$. Partnerships would then form among geographic neighbors, rather than endowment neighbors. If initial endowments are homogeneous, then expanding access can lead to more diverse contacts that change over time but if they are heterogeneous then expanding access can lead to further stratification. This supprts the general claim that expanded access does not always lead to a more equal distribution of resources or to greater sharing. Equality of information access is also contingent on closing the initial gap.

If all agents had equivalent endowments, connections would not become exclusive. Even if networks were to form, the test for instability in Proposition 4 would give $e_n(0) - e_{n+1}(0) = 0$ with a transition time of zero for new connections. Thus the initial conditions strongly influence the shape of subsequent communication. The network stability condition also helps to reinforce the initial assumptions of the model regarding instrumental sorting of agent endowments. If endowments are relatively close, the model shows that networks are endogenously unstable and thus any confusion over sorting order has little effect on subsequent behavior; it only delays one or another resource interaction. On the other hand, if endowments are not close, the gap endogenously determines the choice of partners and the growing separation that follows. Strongly heterogeneous endowments, however, are unlikely to be confused.

A.2. Example: Principle of Collective Fortunes

Networks benefit from increased sharing among partners and a network can sometimes overcome a learning rate disadvantage with a sharing rate advantage. Prior to universal access, let search costs, contracting delays, or barriers to integration separate the independent networks of i and i' and let each network have identical resources and connections, i.e. for each member j   n(i), ∃ j'   n(i') with $e_j(0) = e_{j'}(0)$. Then ∃ $\lambda' < \lambda$ and ∃ $\sigma' > \sigma$ such that $e_{i'}(t) > e_i(t)$ as t -> 8 and similarly for other members of both networks. Thus *a fortiori* with $\lambda' = \lambda$, resources are always greater when $\sigma' > \sigma$.

For illustration, let $\lambda = \sigma = \delta = 1/7$ and C = 3. Then the total private resources of network n(i) grow according to

$$e_{n(i)}(t) = \frac{e_{n(i)}(0)}{8}\left(E^{t/7} + E^{-3t/7}\right).$$

For the network of i', set $\lambda' = 1/8$, $\sigma' = 3/4$, $\delta' = \delta$ and C' = C. Then clearly $\lambda' < \lambda$ and $\sigma' > \sigma$ but the total private resources grow according to

$$e_{n(i')}(t) = \frac{e_{n(i')}(0)}{4}\left(\frac{3}{11}E^{13t/56} + \frac{8}{11}E^{-8t/56}\right).$$

Since, for the larger exponent 13/56 > 1/7 = 8/56, the second network will grow faster as t becomes large although $\lambda' < \lambda$ provides an initial advantage to the first network. This might also be used to justify investments in information technology, which could increase the intrinsic sharing rates of groups. This example also illustrates a second advantage from direct sharing: groups can achieve growth potential that individuals cannot and the output of information collaborators signifantly exceeds that of information soloists. In the first network, $\lambda = \delta$ implying that, in the absence of sharing, total resources could not increase. In the second network, $\lambda' < \delta'$ so that in the absence of sharing total resources would actually decline. Moreover, transfering all resources to the same agent would also fail to achieve substantially greater output. In the absence of a network, $\sigma = 0$ and private resources grow according to $e_i(0)E^{l-d}$. Transfering a group's initial resources would result in $e_{n(i)}(0)E^{l-d}$ implying that for $\lambda < \delta$, resources decay over time. Thus collective information growth is not merely the sum of individual private growth; collective sharing fundamentally alters the pattern of outcomes in the context of a nonrival resource. This model might serve, in part, to explain why the information resources of knowledge workers are difficult to separate from the workers. In this model, the loss of partners results in a loss of output so that alternatives that merely consider a transfer of asset ownership might fail to account for reductions in output with reductions in staffing.

A.3. Example: It's not just what you know but whom you know

Association with high or low value networks provides evidence of this well known folk maxim. Define an "unqualified" agent i relative to network n as one which has poorer resources than the first agent in the next lower ranking network or $e_i < e_{g(n+1)}$. Also, define an "overqualified" agent as one which has richer resources than the last agent of the next higher network or $e_i > e_{g(n-1)+C}$. Correspondingly, a qualified agent relative to network n is one such that i's private endowment falls between the boundaries established by the top and bottom of the surrounding lower and higher networks respectively, that is $e_{g(n-1)+C} > e_i > e_{g(n+1)}$. Then it is possible to show formally that over time an uqualified agent can become qualified and also that an overqualified agent can become qualified since resources converge by Proposition 4.

For a specific example, let there be two separated populations prior to universal access with I = {{40, 30, 20, 1}, {10, 4, 3, 2}}, C = 3, and $\lambda = \sigma = \delta = 1/2$. These parameters yield growth equations similar to those of the first example with $e_{n=1}(0) = 91$ and $e_{n=2}(0) = 19$. Initially, agent eight with $e_8(0) = 1$ is underqualified for his network and agent four with $e_4(0) = 10$ is overqualified for her network. If universal access were to appear after 10 periods, agents might not swap networks and no new interaction necessarily occurs. The private resources of agent eight are roughly $e_8(10) = 1688.05$ while those of agent four are only $e_4(10) = 352.52$. Combined with pooled resources, agent four has only $R(4, 10) = e_4(10) + z_{n(4)}(10) = 1057.48$, well below the private resources of agent eight. The effective time zero ordering has been substantially altered by access (or denied access) to different endowments.

Despite mismatched qualifications at time zero and the implication that agents are not with their network peers, no party prefers to behave differently. If a network were to reject an unqualified candidate meeting the conditions of the proposition in hopes of luring the qualified agent with the introduction of access 10 periods later, the members would have forgone the contributions of agent eight during this time while failing to get a more qualified candidate once the time had passed. Here, agent eight is underqualified initially but qualified in the end. Similarly, if an overqualified agent were to forego the less attractive network meeting the stated conditions, she would have lost the value of the shared pool and found herself less qualified after the time had elapsed. Here, agent four is overqualified initially but qualified in the end. Example three yields a kind of inertia in the sense that early membership or non-membership in the elite can be self perpetuating regardless of initial endowment.

An alternate interpretation prevails if agent eight were poor in information but rich in other resources. If it were possible to subsidize partnership for sufficiently long, an unqualified agent might become qualified by buying access to an information rich network although possibly at the expense of displacing an otherwise qualified agent. The subsidy creates an externality by bumping another agent. The same formulation shows, however, that if the displaced agent moves to a network with roughly equivalent resources then the instability condition suggests that the externality is minimal; contact with these resources is delayed rather than eliminated. Example one showed this effect for $e_n(0) - e_{n+1}(0) \rightarrow 0$.

More generally, these results confirm Proposition 4. By period 10, the private resources of the richest agent, which were $e_1(0) = 40$, are now $e_1(10) = 1688.32$ while those of the poorest agent (now agent 7) were $e_7(0) = 2$ and are now $e_7(10) = 352.46$. Agents one and eight, who are part of the same network, have a ratio $e_1(10)/ e_8(10)$ ♠ 1.00 which is converging to 91/91 while agents one and seven, who are not part of the same network, have a ratio $e_1(10)/ e_7(10)$ ♠ 4.79 which is converging to 91/19.

B. Cases and Real World Applications

A case study comparing regional economies, in particular, illustrates the benefit of increased sharing. Beginning in 1975, California's Silicon Valley and Massachusetts' Route 128 region employed roughly the same number of people but over the next fifteen years Silicon Valley generated three times as many net new technology jobs (Saxenian, 1994). Moreover, between 1986 and 1990, the market value of the Silicon Valley firms increased by more than $25 billion as compared to $1 billion along Route 128. Saxenian argues that information sharing and collaboration account for most of this difference, with three factors emerging as explanatory variables. First, a higher level of vertical integration in New England firms reduced information transfers between markets and business units. Second, more defense funding led to a premium on secret research which could not be shared. Third, engineering and technical expertise moved more freely in California's open and spirited environment. These forces greatly increased the volume of information sharing in Silicon Valley, subsequently compounding regional wealth. Firms in Massachusetts more aggressively sought to protect their intellectual capital advantage (as in Proposition 3) yet ultimately had less to protect (as in Proposition 6).

Examples of advantage secured through a network of connections are also not hard to find. In a fourteen year national longitudinal study of high school graduates, [James, '88], researchers found that the selectivity of one’s alma mater increased earnings even after controlling for SAT scores and academic performance. A similar study confirmed an inertial effect insofar as certain minority populations benefited relatively more in increments to their incomes than non-minority populations did when they attended high quality universities (Daniel et al., 1995). Connections established in college can often provide a sustaining advantage later in life. As modeled here, access to information rich networks for short periods becomes a valuable asset in information trading environments.

VIII. Implications

A. Relaxing the Nonrival Assumption

Although much of the information sharing behavior observed in the propositions rests on the nonrival property, relaxing this assumption yields relatively consistent results. Sharing might dilute information's opportunities for use limiting its nonrival property. This might be the case, for example, if sharing several market tips led others to act on an arbitrage opportunity rendering it unusable to the original source. Dilution is different from temporal depreciation, referring instead to reduced opportunity for use based on how many others are using it. If information dilutes at rate when it is shared between i and j, then i receives $e_i(t+\Delta t) = (1-\sigma)e_i(t) + \sigma(1- \ )e_i(t) + \sigma(1- \ )e_{jt}$ and similarly for j. These terms represent i’s residual unshared information, i’s residual shared but diluted information, and reciprocal sharing from j respectively. From i’s perspective, trade ensues only if gains $\sigma(1- \ )e_{jt}$ exceed losses $\sigma \ e_i(t)$. Rearranging this inequality gives $e_j(t) > \frac{}{1-} e_i(t)$, that is i will only share with agents j above some minimum relative threshold that increases in the dilution rate. According to this formula, for $\ > \frac{1}{2}$, agents will only share with those who are more elite than themselves. Sharing halts for precisely the same reason as in the Information Climber Effect. The most elite agent has no potential trading partners and opts out of the system, an effect that cascades throughout the population. For dilution above $\frac{1}{2}$, sharing should also halt because it results in a net loss of resources -- the benefactor loses more than a beneficiary gains. Thus, the propositions hold up to the point where information starts to behave like a tangible good, the point where traditional goods intuitions operate. Assuming that $\ < \frac{1}{2}$, the propositions roughly hold with the proviso that agents falling below the relative threshold $e_j(t) = \frac{}{1-} e_i(t)$

become ineligible for information trading.[18] Networks still form among relative peers. This ratio also implies that certain channels could go unused if there are too few eligible partners. A policy which boosts the initial endowments of ineligible agents then has a considerable effect on their true levels of access. **B. Parameter Changes**

The model makes several predictions, particularly in terms of how information technology affects access A, channels C, sharing σ, and resources $e_i(t)$. In this model, increasing access parameter A can lead to compounding winner-take-all effects and a secession of the information elite in the context of initial inequality. Proposition Two specifically shows the effects of increasing access defined as increasing reach within the population. IT can also be used to increase the number of channels C available to each agent. Supplying every member of the population with an equal number of channels does not necessarily eliminate resource differences as suggested by Propositions One and Four although it can help increase network sizes and it marginally increases network instability. Importantly, however, C is also a proxy for bounded rationality or limited attention. Information technology is unlikely to permit individuals to widen their focus to the entire population. "You can [have access] to hundreds of millions, but you can't know them all because all you can remember is 3000. All you can do is replace the label 'physical acquaintance' with 'virtual acquaintance'" (Rowe, 1996, from an interview with M. Dertouzos).

Of the parameters responsible for resource differences, initial endowments e_i are among the most critical. Since the rich tend to become richer over time, this model suggests that shrinking initial gaps $e_i(0) - e_j(0)$ can reduce subsequent gaps. One policy implication is that it might be possible to close these gaps through an improved incentive system for public disclosure. If agents disclose their private information, then others might enjoy improved resources in subsequent periods. As suggested by Proposition Three, however, the difficulty may lie in overcoming relative objective functions. Agents who privately value their relative advantage may refuse to share.

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[18]Consider that for ? = 1/2, $e_j(t) = e_i(t)$, for ? = 1/3, $e_j(t) = (1/2)e_i(t)$, and for ? = 0, $e_j(t) = (0)e_i(t)$ implying that j need have no resources to be an eligible partner.

To the extent that IT can increase either the learning or sharing parameters, λ or σ respectively, the model suggests that information resources grow faster. In the long run, the learning parameter most strongly affects growth rates but, as the second example suggests, situations exist in which improved resource sharing can expedite resource improvement. IT may be much more effective changine a group's intrinsic σ than its λ, thus the model helps motivate groupware, data warehouses, and corporate knowledge bases on the basis of improved sharing through information systems. These findings are also consistent with descriptions of resource growth in organizations (Huber, 1991; Jarvenpaa & Ives, 1994; Quinn, 1992) through sharing.

(Clemons, Reddi & Row, 1993; Malone, Yates & Benjamin, 1987)Conceivably, information technology could have an effect opposite to that of helping the rich to get richer if it curtailed effective access.[19] Noise might increase as broadcast communication and spamming[20] become ubiquitous. Information overload and confusion might confound attempts to locate new and better partners, forcing searchers to pursue locally available options. Noise reduces access and reverses the effects of Propositions One, Two, and Four. Given collaborative filtering, increasingly powerful search engines, and kill files, however, the trend toward increasing access seems likely to dominate the effects of increased information noise.

IX. Conclusions

Although channel access can be created equal, information resources are not necessarily shared this way. In general terms, the model suggests that decreasing search costs and increasing access can increase stratification if initial endowments start out unequal and growth rates are high. The model offers two reasons for this effect.

The principal reason is that "bounded rationality" or capacity constraints dampen the impact of universal access. Until and unless infrastructure, including human processing capacity, permits unbounded

[19]We are indebted to John Sterman for this observation.

[20]This is the practice of posting irrelevant material to large electronic communities in the hope of generating even a few intended respondents. The senders typically do not bear the nuisance costs they impose on the broader audience they inconvenience.

resource transfers, agents will find it privately advantageous to focus their attention on the richest information sources. In particular, richer agents might limit their attention to one another. Facing a glut of information, agents can find value in selective focus. Relatively closed networks then form, not from planned exclusivity but in response to the problem of dealing with information and contact overload. Thus policies which increase information overload may amplify problems of selective focus and network closure and segregation.

Not all information resources are public, and the heterogeneity of private resources also contributes to increased disparity. This effect builds on itself as heterogeneity tends to increase with high growth rates over time. It is also consistent with increasingly privatized information (Branscombe, 1995). Anyone wishing to contact a private source faces increased difficulty as ubiquitous communications allows others to know of and seek the same resource. This can bid up the price of information and experience with the ironic consequence that increased access can lead to decreased availability. Both elements of this analysis show that universal access in a voluntary exchange economy can inhibit rather than facilitate equality, with the information rich becoming information richer.

Ancillary results of the model imply that (1) sharing can halt if agents value relative differences over absolute totals, (2) improved access can lead to more winner-take-all markets in the context of broadcast, (3) nonrival resources can grow at increasing rates, (4) information collaborators can overtake information soloists, and (5) differences in channel access can be self perpetuating. The model also conforms relatively well to several empirical observations including segmentation on the Internet, sharing incentives within, resource growth through sharing, and advantage secured through networks of connections.

X. Glossary of Variables

Symbol	Description
A	Access, the total number of agents that can be located or reached at any given time; 1 = A = I.
C	The total number of channels available to an agent. Distinct from A, this is the number of mutual connections that it is possible to hold simultaneously; C < I - 1.
δ	Depreciation rate, obsolescence of information resources with time.
	Dilution rate, reduction in nonrivalry assuming loss through sharing.
$e_i(t)$, $e_{-i}(t)$, $e_{n(i)}(t)$	Endowment of agent i at time t; shorthand for all endowments in i's network at time t other than agent i's or $\sum_{j \in n(i),\ j^\circ i} e_j(t)$; shorthand for all endowments in i's network at time t or $\sum_{j \in n(i)} e_{jt}$.
$\overline{e}$	The average or expected endowment.
f(i)	Function giving the network index of agent i.
g(n)	Function returning highest ranking agent of network n.
i, j, k {1, 2, ... I}	Individuals, firms, or agents; the total number is I.
i'	A replacement agent for agent i.
L_{ij}	Link, strength of interest in i's connection to j. L_{ij} [0, 1]
λ	Learning parameter; the rate at which information resources grow.
n(i)	The set of agents in i's network.
Φ(i, t, I)	Any concave production technology with positive output on the resources of i, R(i, t, I) or $\Phi > 0$, output increasing in input $\Phi' > 0$, and diminishing returns $\Phi'' = 0$.
R(i, t, I)	Information resources of i at time t, defined as the sum of $e_i(t)$ and other $\sigma e_j(t)$ to which i has access in population I.
σ	Sharing parameter; the fraction of the private endowment disclosed to other agents.
$z_n(t)$	Shorthand for the shared resources of a network at time t equal to $\sigma[e_1(t) + e_2(t) + \ldots e_{C+1}(t)]$

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XII. Mathematical Appendix of Proofs

For convenience, each proposition is repeated below together with its accompanying proof.

Proposition 1.A (*Principle of Growing Network Inequality*): Utility maximization in voluntary networks increases the resources of the information elite i.e. $\forall$ i < j in period 1, R(i, 1) - R(j, 1) = R(i, 0) - R(j, 0). The following strategy is a Nash equilibrium:

$$\text{strategy(i)} = \begin{Bmatrix} L_{ij} = 1 & f(i) \geq f(j), i \neq j \\ L_{ij} = 0 & f(i) < f(j) \end{Bmatrix}$$

implying that agent i prefers to communicate with other agents having equal or greater endowments than those in network f(i).

Proof: We prove proposition 1 by showing that a voluntary Nash equilibrium strategy exists, that this strategy maximizes the payoff of each agent from the choices available, and that the elite reap a greater share of benefits.

Step 1: The above elite strategy is a Nash equilibrium. Agent i chooses L_{ij} such that this strategy provides every agent with no more than C network partners and the last chosen partner is at least as elite as any other partner available to this agent. There are three cases, assuming i ? j

(a) f(i) > f(j) -- By the definition of f(), each such agent j has a richer endowment than i but by the equilibrium strategy agent j chooses $L_{ji} = 0$ so i cannot connect to j since $L_{ij}L_{ji} = 0$. Deviation by choosing L_{ij} [0, 1) will not change this outcome.

(b) f(i) < f(j) -- By the definition of f(), each agent i has a richer endowment than any such j. Since all other agents k for which f(i) = f(k) have richer endowments, i finds it costly to fill any channel with communications to j, thus $L_{ij} = 0$ strictly dominates L_{ij} (0, 1].

(c) f(i) = f(j) -- In equilibrium, agents j choose $L_{ji} = 1$ and by the definition of f(), there are exactly C such agents other than i. Choosing $L_{ij} = 1$ therefore strictly dominates $L_{ij} = 0$ since either the channel goes unused or agent i links to agents k where R(k,1) < R(j,1). If L_{ij} (0, 1) then agent i is not getting the maximum value from the channel.

Step 2: The elite strategy maximizes information access from the choices available to each agent i.e. $\nexists$ j, k such that R(i',1) > R(i,1) where n(i') = n(i) ≈ {k} \ {j} and $L_{ki} = 1$. Before any change, information access is given by

$$R(i,1) = e_i(1) + s \sum_{j \in n(i), j \neq i} L_{ij} L_{ji} e_j(1)$$

switching k for j provides information access $R(i',1) = R(i,1) - \sigma L_{ij}L_{ji}e_j(1) + \sigma L_{ik}L_{ki}e_k(1)$. If R(i',1) > R(i,1) then $\sigma L_{ik}L_{ki}e_k(1) > \sigma L_{ij}L_{ji}e_j(1)$ but for $k < g \circ f(i)$, $L_{ki} = 0$ so $\sigma L_{ik}L_{ki}e_k(1)$ adds nothing while losing a share of $e_j(1)$. Thus $R(i',1) < R(i,1)$. For $k > g \circ f(i) + C$, $e_k(1) < e_j(1)$ so the switch is also a net loss. In the case of $g \circ f(i) = k = g \circ f(i) + C$ agent k is already in the network.

Step 3: $\forall$ i < j in period 1 , $R(i,1) - R(j,1) = R(i,0) - R(j,0)$. Prior to network formation $R(i,0) = e_i(0) > e_j(0) = R(j,0)$ since $e_i(0) > e_j(0)$. After network formation, total resource access is provided by

$$\boxed{R(i,1) = e_i(0) + s \sum_{k \in n(i),\, k \neq i} L_{ik} L_{ki} e_k(0)}$$

$$\boxed{R(j,1) = e_j(0) + s \sum_{k \in n(j),\, k \neq j} L_{jk} L_{kj} e_k(0)}$$

By the definition of f(), $e_{g \circ f(i)}(1) > e_{g \circ f(i)+C}(1) > e_{g \circ f(j)}(1) > e_{g \circ f(j)+C}(1)$ and the more elite agent has access to richer endowments which proves proposition 1.A.

Proposition 1.B (*Principle of Growing Equality Within Networks*): Preexisting differences in resource access among agents in the same network decrease. That is for f(i) = f(j), $R(i, 1) - R(j, 1) = R(i, 0) - R(j, 0)$.

Proof: This is a straightforward comparative static result. Prior to network formation, $R(i,0) = e_i(0)$ and likewise for j so that $R(i,0) - R(j,0) = e_i(0) - e_j(0)$. By Proposition 1.A, all L_{ik} are either 0 or 1 so they may be suppressed. Assuming that agents i and j are in the same network, i.e. that f(i) = f(j), then following network formation, agent i has resources $R(i,1) = e_i(0) + \sigma e_{-i}(0)$ and likewise for j where $e_j(0)$ $e_{-i}(0)$ and $e_i(0)$ $e_{-j}(0)$. Since members n(i) = n(j), resource differences converge, $R(i,1) - R(j,1) = [e_i(0) + \sigma e_{-i}(0)] - [e_j(0) + \sigma e_{-j}(0)]$ which simplifies to $[e_i(0) + \sigma e_j(0)] - [e_j(0) + \sigma e_i(0)] = [e_i(0) - e_j(0)](1 - \sigma)$. Since σ [0,1], the resource gap between agents of the same network shrinks for all σ above complete secrecy.

Corollary 1 -- The elite have a greater choice of partners. Proceeding inductively, the initial case for any agent i from 1 through C+1 is the choice set equal to $|\{1, 2, ... I\} \setminus \{i\}|$. In general, the size of the choice set is $|\{1, 2, ... I\} \setminus \{i\}| - (C+1)[f(i) - 1]$ and for agent i > C+1, the second term is > 0 and increasing. Subsequent agents must therefore have choice sets no larger than more elite agents.

Proposition 2 (*Winners-Take-All Effect of Increased Access*): Given a capacity constraint, increasing access raises the probability that more elite agents become more attractive partners in any reachable network. More precisely, let B represent the event that j is an attractive partner indicating that other agents choose to contact j. Then p(B | j = C) -> 1 and p(B | j > C) -> 0 as access approaches the population size A -> I.

Proof: Let j represent an arbitrary agent in the collection of I agents. Then, the probability that j is in the first C choices of a sample of size A is given by the number of combinations involving agents above and below j:

$$\frac{\sum_{m=1}^{C}\binom{j-1}{m-1}\binom{I-j}{A-m}}{\binom{I}{A}}$$

Where the expression $\binom{N}{K}$ is undefined for K > N, the above expression is equivalent to a summation with changed bound

$$\frac{\sum_{m=1}^{j}\binom{j-1}{m-1}\binom{I-j}{A-m}}{\binom{I}{A}}$$

The summation in this numerator, however, simplifies to the reduced form expression $\binom{I-1}{A-1}$ which, when divided by $\binom{I}{A}$ reduces to $\frac{A}{I}$. Thus the probability of reaching any of the first C agents is a linearly increasing function of A and p(B | j = C) -> 1. On the other hand, as A -> I the righthand combinatorial choice in the numerator becomes undefined so that p(B | j > C) -> 0.

Proposition 3 (*Information Climber Effect*): If agents maximize relative assets in lieu of absolute assets, information sharing halts. That is, the following strategy becomes a Nash equilibrium:

$$\text{strategy (i)} = \begin{Bmatrix} L_{ij} = 1 & j < i \\ L_{ij} = 0 & j > i \end{Bmatrix}$$

Proof: Maximizing relative assets implies that agents seek to increase the gap between their assets and those of others. Rather than absolute maximization, the objective becomes

$$\boxed{\max_{L_{ij}} u(\sum_{j \neq i}^{I} L_{ij} L_{ji} s e_j(t) - e_i(t))} \text{ such that } \boxed{|n(i)| \leq C+1} \text{ and } \boxed{L_{ij}, L_{ji} \in [0,1]}$$

This objective function breaks into two components representing agents with richer and poorer endowments respectively. Agent i seeks to increase the advantage over less elite agents and to decrease the advantage of more elite agents. Recalling that i < j implies $e_i(0) > e_j(0)$ this yields

$$\boxed{\max_{L_{ij}} u\left(\sum_{j<i} L_{ij} L_{ji} s\, e_j(t) - e_i(t) + \sum_{j>i} L_{ij} L_{ji} s\, e_j(t) - e_i(t)\right)}$$

Consider the first agent i = 1. Under the conditions of the second summation, resources after full sharing with any agent j would be such that $e_i(1) = e_i(0) + \sigma e_j(0)$ and $e_j(1) = e_j(0) + \sigma e_i(0)$ but for all agents j > i it must be the case that $e_j(0) < e_i(0)$ and the relative gap becomes $e_i(1) - e_j(1) = [e_i(0) + \sigma e_j(0)] - [e_j(0) + \sigma e_i(0)] = (1 - \sigma)[e_i(0) - e_j(0)]$. This gap at time 1 is smaller than the gap at time 0 unless agent i sets $L_{ij} = 0$. Although agent j may set $L_{ji} = 1$, agent 1 can do no better by sharing with any less elite agent. For the first agent, the first summation is zero since there are no agents j < i. But then the highest ranking agent communicates with no one, leaving agent 2 in the position previously enjoyed by agent 1. By induction, no agent communicates with less elite agents and sharing halts. Also, since u"(e) < 0, agents do not collude. That is, the loss from losing ground to a less elite agent exceeds the gain from overtaking a more elite agent.

Lemma 4 -- For private resources, the resource growth equations in continuous time are given by:

$$e_i[t] = e_i[0]E^{t(l-s-d)} + \frac{e_{n(i)}[0]}{c+1}\left[-E^{t(l-s-d)} + \frac{1}{2r}\left((l-s+r)E^{\frac{t}{2}(l-s-2d+r)} - (l-s-r)E^{\frac{t}{2}(l-s-2d-r)}\right)\right]$$

and for pooled resources by:

$$z_n[t] = \frac{s e_{n(i)}[0]}{r}\left[E^{\frac{t}{2}(l-s-2d+r)} - E^{\frac{t}{2}(l-s-2d-r)}\right]$$

Proof -- The instantaneous rate of increas in private resources in the learning that occurs from access to priate and pooled resources is $\lambda(e_i[t] + z_{n(i)}[t])$ while the instantaneous rate of decrease is given by the sharing into the common pool and the depreciation rate $-(\sigma + \delta)e_i[t]$. Similarly, the change in pooled resources increases by the sharing from private resources and decreases by depreciation giving rise to the differential equations:

$$e_i'[t] = (l-s-d)e_i[t] + l z_{n(i)}[t]$$

$$z_{n(i)}'[t] = s\left[e_1[t] + e_2[t] + \ldots e_{c+1}[t]\right] - d z_{n(t)}[t]$$

In matrix form, these equations can be rewritten for the collective network as

$$\begin{bmatrix} e_1' \\ e_2' \\ \vdots \\ e_{c+1}' \\ z_{n(i)}' \end{bmatrix} = \begin{bmatrix} (l-s-d) & 0 & \cdots & 0 & l \\ 0 & (l-s-d) & \cdots & 0 & l \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & (l-s-d) & l \\ s & s & \cdots & s & -d \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{c+1} \\ z_{n(i)} \end{bmatrix}$$

Note that the depreciation term is an additive constant across the main diagonal -- its effect will thus be minimal. The eigenvectors will be identical to those of a matrix with no depreciation term while the eigenvalues will simply be offset by -δ. The matrix has only three unique eigenvalues with $v_1 = (\lambda - \sigma - \delta)$, $v_2 = \frac{1}{2}(\lambda - \sigma - 2\delta - r)$, $v_3 = \frac{1}{2}(\lambda - \sigma - 2\delta + r)$, and with accompanying eigenvectors

$$\begin{array}{cccccc} \{v_1, & v_1, & \ldots & v_1, & v_2, & v_3,\} \\ \{\{1,-1,0,\ldots 0,0\}, & \{1,0,-1,0,\ldots 0,0\}, & \ldots & \{1,0,\ldots 0,-1,0,0\}, & \left\{\frac{2l}{c_2},\ldots\frac{2l}{c_2},-1\right\}, & \left\{\frac{2l}{c_3},\ldots\frac{2l}{c_3},-1\right\}\} \end{array}$$

These equations introduce a few constants to simplify notation. These are $r = \sqrt{(l+s)^2 + 4cls}$, $c_2 = (\lambda - \sigma + r)$, $c_3 = (\lambda - \sigma - r)$, which are roughly v_3 and v_2 without depreciation δ. Since c, λ, $\sigma = 0$, it follows that $c_2 = 0$ and $c_3 = 0$ always. These facts will simplify subsequent expressions. Putting the eigenvalues in a vector V and the eigenvectors as the columns of a matrix S, we can calculate the time varying solution from the initial conditions as $S \bullet \text{Exp}[t^*V] \bullet S^{-1} \bullet [e_1[0], e_2[0], \ldots e_{c+1}[0], z_n[0]]$ where the initial conditions for the pooled resources of the network $z_n[0] = 0$ and as before, $e_n[0]$ abbreviates the sum of initial resources $e_1[0] + e_2[0] + \ldots e_{c+1}[0]$. This multiplication simplifies to

$$e_i[t] = e_i[0]E^{tv_1} + \frac{e_n[0]}{c+1}\left[-E^{tv_1} + \frac{1}{2r}\left(c_2 E^{tv_3} - c_3 E^{tv_2}\right)\right]$$

$$z_n[t] = \frac{s e_n[0]}{r}\left[E^{tv_3} - E^{tv_2}\right].$$

Then substituting for v_1, v_2, v_3, c_2, and c_3 provides the required results. In these equations, v_1 can be roughly interpreted as affecting individual contributions, whereas v_2 and v_3 can be interpreted as the changes contributed by group processes.

Proposition 4 (*Convergence and Stability Conditions*) -- For any two agents i and j, $e_i[t]/e_j[t] \to e_{n(i)}[0]/e_{n(j)}[0]$ subject to the stability condition

$$E^{tr} < \frac{\left[(e_n - e_{n+1})(l - s - r) - 2s(c+1)e_{n+1}\right]}{\left[(e_n - e_{n+1})(l - s + r) - 2s(c+1)e_{n+1}\right]}.$$

Since the stability condition requires growth in *private* resources, it also follows that for stable networks, if f(i) = f(j), then $e_i[t]/e_j[t] \to 1$ whereas for f(i) < f(j) then $e_i[t]/e_j[t] \to K > 1$ and $e_i[t] - e_j[t] \to 8$.

Proof: Lemma 4 gives the closed form analytic defintions for $e_i[t]$ and $z_n[t]$ over time. The stability condition can be defined as the point where the private resources of an agent in i's network, n(i), no longer exceed the combined private and pooled resources in the next network. To anyone outside the network, both private and pooled resources from another network would be attractive gains. Thus, the test is $e_{j_n}(t) < e_{j_{n+1}}(t) + z_{j_{n+1}}(t)$ or, after separating private and pooled resources, $e_{j_n}(t) - e_{j_{n+1}}(t) < z_{j_{n+1}}(t)$.

The gap between the initial endowments $e_i[t]$ and the average for j's own group $e_n[0]/(c+1)$ is likely to be negligible relative to growth in pooled resources. So, treating the first two terms in the definition of private resources as if they cancel, expanding the stability condition yields:

$$\boxed{\frac{1}{2r(c+1)}\left[(e_n - e_{n+1})\left(c_2 E^{tv_3} - c_3 E^{tv_2}\right)\right] < \frac{s}{r}\left[e_{n+1}\left(E^{tv_3} - E^{tv_2}\right)\right]}$$

After cross multiplying the denominators and cancelling *r*'s, the exponential terms can be separated.

$$\boxed{\left[(e_n - e_{n+1})c_2 - 2s\,(c+1)e_{n+1}\right]E^{tv_3} < \left[(e_n - e_{n+1})c_3 - 2s\,(c+1)e_{n+1}\right]E^{tv_2}}$$

Dividing through by the left hand side coefficient and the right hand side exponential cancels much of the exponent and yields a final expression for testing stability.

$$E^{t(v_3 - v_2)} = E^{tr} < \frac{\left[(e_n - e_{n+1})c_3 - 2s\,(c+1)e_{n+1}\right]}{\left[(e_n - e_{n+1})c_2 - 2s\,(c+1)e_{n+1}\right]}.$$

This division requires dividing by two separate terms. The exponential will always be positive and thus never affects the inequality. The denominator in the stability test, however, may sometimes be negative, which reverses the sense of the inequality. Each variable is positive and as noted in Lemma 4, $c_2 = 0$ always so the actual sense of the inequality will depend on the relative sizes of $(e_n - e_{n+1})c_2$ and $2\sigma(c+1)e_{n+1}$. By assumption, $(e_n - e_{n+1}) > 0$ so the denominator becomes negative when the gap between initial network resources is small relative to the total initial resources e_{n+1} in the next network or when the number of connections c is very large. Note also that $c_3 = 0$ always so that the numerator is strictly negative. If the denominator is positive the test fails permanently because E^{tr} cannot be less than zero. Networks are permanently stable. If the denominator is negative, however, the inequality reverses and the test must eventually pass any positive threshold because time does not appear on the right hand side. Networks in this case are unstable.

Proposition 5 -- (*Principle of Convex Growth*) Let $\Phi(e_t)$ be any concave production technology such that $\Phi' > 0$ and $\Phi'' < 0$ i.e. output increases with input but exhibits decreasing returns to scale. Assume also that $\Phi(e_t) > 0$ so that Φ does not produce negative output. Let $\Psi(t \mid e_0)$ be the information state describing the available information when Φ is the production technology. Then Ψ is a convex function of time with both $\Psi' > 0$ and $\Psi'' > 0$. Since concave technology is the most difficult such case, *afortiori*, any nonnegative, nondecreasing production technology Φ also gives rise to a convex information state Ψ regardless of its convexity properties.

Proof: The assumptions on production technology that $\Phi > 0$, $\Phi' > 0$, and $\Phi'' < 0$ are standard in economics. The nonrivalry of an information good then ensures that $\Phi(e_t)$ is the production *rate*, i.e. that all inputs to production are recovered after production in addition to whatever is produced. Thus if $\Psi(t \mid e_0)$ represents the available information, then changes in information are $\Psi'(t \mid e_0) = \Phi(\Psi(t \mid e_0))$. By differentiation, $\Psi''(t \mid e_0) = \Phi'(\Psi(t \mid e_0))\Psi'(t \mid e_0) = \Phi'(\Psi(t \mid e_0))\Phi(\Psi(t \mid e_0))$ where both $\Phi > 0$, $\Phi' > 0$. But this shows that both $\Psi' > 0$ and $\Psi'' > 0$.