Optimal Bidding in Online Auctions

Annual eBusiness Conference

D. Bertsimas, J. Hawkins and G. Perakis
Sloan School, MIT

April 2002
1 Outline

- Introduction
- Data and Formulation
- Single Auction
- Multiple Auctions
- Conclusions

2 eBay Overview

- Online auctions important and distinctly new applications of internet
- Consumer to Consumer Internet auction house
- eBay has over 18.9 million registered users and over $5 billion worth of goods in 2000 in over 4500 categories
- eBay auction is similar to a second-price sealed bid Vickrey auction

2.1 Data available

- Finite duration (3, 5, 7, or 10 days)
- Items description
- Number of bids
- ID of all the bidders and time of bid, but not the amount (available after the end)
- ID of current highest bidder
- Time remaining until end of auction
- Whether or not the reserve price has been met
- Starting price of the auction
- Listed price (Second highest price)

2.2 Characteristics

- Vickrey auction
- Bidders react to the indirect behavior of other bidders through observation of the listed price
- A lot of activity near the final seconds (sniping)
- With a fixed length auction, a submitted bid will be accepted with a probability $p$ in the final seconds.
2.3 Objective philosophy

- Determine optimal bidding strategy for a single and multiple simultaneous or overlapping online auctions
- Model requirements:
  - captures the essential characteristics of online auctions
  - leads to a computationally feasible algorithm that is directly usable by bidders
  - parameters for the model can be estimated from publicly available data
- Dynamic optimization approach

3 Single item auction

3.1 State

- Length of auction is discretized into $T = 13$ periods: 5 days, 4 days, 3 days, 2 days, 1 day, 12 hours, 6 hours, 1 hour, 10 minutes, 2 minutes, 1 minute, 30 seconds, and 10 seconds remaining
- **State:** $(x_t, h_t)$ for $t = 1, \ldots, T + 1$
  \[
  x_t = \text{listed price at time } t \\
  h_t = \begin{cases} 
  \text{the agent's proxy bid if the highest bidder at time } t \\ 
  0, \text{otherwise} 
  \end{cases}
  \]
- Shorthand: $w_t = 1$ if the agent has a proxy bid, 0 otherwise

3.2 Control + Randomness

- **Control:**
  \[
  u_t = \begin{cases} 
  \text{amount the agent bids at time } t, \text{ where} \\ 
  w_t x_t \leq u_t \leq A 
  \end{cases}
  \]

- **Randomness:**
  \[
  q_t = \text{amount population bids at time } t \\
  \tilde{h}_t = \text{unknown proxy bid at time } t \\
  v_t = \text{indicates if bid submitted in period } t \text{ is accepted} \\
  P(v_T = 1) = p
  \]

3.3 Dynamics

- $w_t = 0 \Rightarrow x_{t+1} = \text{second highest bid from } q_t, u_t \text{ and } \tilde{h}_t$
- $w_t = 1 \Rightarrow x_{t+1} = \text{second highest bid from } q_t, u_t$
- $\tilde{h}_{t+1} = \begin{cases} 
  u_t \text{ if the agent was the highest bidder at time } t \\ 
  0, \text{otherwise} 
  \end{cases}$
<table>
<thead>
<tr>
<th>Strategy</th>
<th>Win %</th>
<th>Utility</th>
<th>$ Spent</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>64.2</td>
<td>26.6</td>
<td>108.6</td>
</tr>
<tr>
<td>Bid A at ( t = 0 )</td>
<td>45.2</td>
<td>18.6</td>
<td>108.9</td>
</tr>
<tr>
<td>Bid A at ( t = T - 1 )</td>
<td>48.8</td>
<td>20.0</td>
<td>108.9</td>
</tr>
<tr>
<td>Bid A at ( t = T )</td>
<td>9.2</td>
<td>3.8</td>
<td>108.2</td>
</tr>
</tbody>
</table>

### 3.4 Objective

Agent wants to maximize the expected utility

\[
\text{maximize } E[U(x_{T+1}, h_{T+1})]
\]

\[
U(x_{T+1}, h_{T+1}) = w_{T+1}(A - x_{T+1})
\]

- agent will not bid for an item beyond his budget \( A \)
- wants to win the auction at the lowest possible price
- he is indifferent between not winning the auction and winning it at the budget \( A \)

### 3.5 Bellman equation

\[
J_t(x_t, h_t) = \max_{\mu_t \in \mathcal{P}(x_t, h_t)} \mathbb{E}[q_t, v_t, h_t \mid J_{t+1}(x_{t+1}, h_{t+1})], \quad t = 1, \ldots, T
\]

\[
= \max_{\mu_t \in \mathcal{P}(x_t, h_t)} \sum_{q=0}^{A} \sum_{v=0}^{1} \sum_{h=x_t}^{A} J_{t+1}(x_{t+1}, h_{t+1})
\]

\[
\cdot P(q_t = q, h_t = h \mid x_t) P(v_t = v)
\]

### 3.6 Parameter estimation

Estimated \( P(q_t, h_t \mid x_t) \)

- Used 1772 completed auctions (22478 bids) for Palm Pilot III over a two week period, whose final selling price was between $70 and $200
- Used 4208 completed auctions (50766 bids) for stamp collections with final prices $100 to $900

### 3.7 Results for PDAs

Bidding for a Palm Pilot III with a budget of $150

### 3.8 Stamp collections

Bidding for a stamp collection with a budget of $500
4 Multiple auctions

- $N$ simultaneous auctions all ending at the same time
- $A_i$ is the budget for auction $i$, total budget $A$
- Utility $U(x_{T+1}, h_{T+1}) = \sum_{i=1}^{N} (A_i - x_{T+1}^i)w_{T+1}^i$
- State for auction $i$ $(x_i^i, h_i^i)$; control $u_i^i$; randomness $(q_i^i, v_i^i, h_i^i)$
- Exact DP is barely tractable even for two auctions
- We use approximate dynamic programming methods

4.1 Integer programming

Integer Programming Approximation (IPA)

Solve auctions independently with adjusted utility function, then for given state can solve approximate cost-to-go.

- $d_i(j)$ the expected utility of bidding $j$ in auction $i$ given state $(x_i^i, h_i^i)$ and optimally bidding in this single auction thereafter
  \[
  d_i^*(j) = E_{q_i^i, v_i^i, h_i^i}[J_i^*(x_i^i+1, h_i^i+1|u_i^i = j)]
  \]
  \[
  J_i^*(x_i^i, h_i^i) = \max_j d_i^*(j)
  \]
- Decision variables:
  \[
  u_i(j, t) = \begin{cases} 
  1, & \text{if the agent bids at least } j \text{ in auction } i \text{ at time } t \\
  0, & \text{otherwise}
  \end{cases}
  \]

4.1.1 Model

\[
\text{maximize} \quad \sum_{i=1}^{N} \sum_{j=0}^{A_i} u_i(j, t)(d_i^*(j) - d_i^*(j - 1))
\]
subject to
\[
\begin{align*}
  & u_i(j, t) \leq u_i(j - 1, t) & \forall i, j \\
  & \sum_{i=1}^{N} \sum_{j=0}^{A_i} u_i(j, t) \leq A \\
  & \sum_{j=1}^{A_i} u_i(j, t) \geq h_i & \forall i \\
  & u_i(j, t) \in \{0, 1\} & \forall i, j
\end{align*}
\]
<table>
<thead>
<tr>
<th>Method</th>
<th>% Won</th>
<th>Utility per Auction</th>
<th>$ Spent per Win</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADP1</td>
<td>29.3</td>
<td>17.6</td>
<td>130</td>
</tr>
<tr>
<td>ADP2</td>
<td>31.5</td>
<td>9.5</td>
<td>140</td>
</tr>
<tr>
<td>IPA</td>
<td>30.3</td>
<td>45.8</td>
<td>99.7</td>
</tr>
<tr>
<td>PIPA1</td>
<td>29.9</td>
<td>46.6</td>
<td>98.1</td>
</tr>
<tr>
<td>ADP1</td>
<td>51.9</td>
<td>487.2</td>
<td>187.1</td>
</tr>
<tr>
<td>ADP2</td>
<td>47.2</td>
<td>429.2</td>
<td>196.9</td>
</tr>
<tr>
<td>IPA</td>
<td>64.0</td>
<td>677.0</td>
<td>147.3</td>
</tr>
<tr>
<td>PIPA1</td>
<td>64.3</td>
<td>686.6</td>
<td>144.3</td>
</tr>
</tbody>
</table>

4.1.2 \textit{N} = 3; Palm Pilots III

4.1.3 \textit{N} = 3; Stamps

4.2 Insights

- IP based methods \textit{(IPA, PIPAa)} clearly outperform ADP methods \textit{(ADP1, ADP2)}.

- When it is computationally feasible to find the optimal strategy \textit{(N = 2)}, IPA is almost optimal. The exact DP leads to slightly higher utility.

- \textit{PIPA1} and \textit{PIPA2} lead to slightly better solutions compared to \textit{IPA}, but at the expense of a much higher computational effort.

5 Conclusions

- Dynamic Optimization offers edge in online auctions

- New effective method to solve DPs using IPs

- Extension to stochastically arriving auctions

- Extensions to a trading system using auctions