Electronic Sales Strategies for Time-Sensitive Markets

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Practical Motivation

• On June 1st, a company replaces 4 identical NC machine-tools with a new production line

• Buyers’ willingness to pay for each varies between $170,000 and $230,000

• On average, potential buyers come up every 3 days

• Future revenues are discounted as historical ROI is 20% p.a.

How should the company sell its old machines?
Example of Sales Strategies

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Company runs a multi-unit auction on eBay for 30 days</strong></td>
<td><strong>Company announces a price of $190,000 for each machine</strong></td>
</tr>
<tr>
<td>• On average, 10 bidders in the auction</td>
<td>• Each sale generates $190k</td>
</tr>
<tr>
<td>• On average, the sale price for each machine will be $199k</td>
<td>• Gross revenue is $760k</td>
</tr>
<tr>
<td>• Gross revenue of 4 x $199k = $798k only available on June 30th</td>
<td>• On average:</td>
</tr>
<tr>
<td></td>
<td>» 1st sale on June 9th</td>
</tr>
<tr>
<td></td>
<td>» 2nd sale on June 18th</td>
</tr>
<tr>
<td></td>
<td>» 3rd sale on June 27th</td>
</tr>
<tr>
<td></td>
<td>» 4th sale on July 5th</td>
</tr>
<tr>
<td>• The Expected Discounted Revenue is $786k</td>
<td>• The Expected Discounted Revenue is $755k</td>
</tr>
</tbody>
</table>
Research Questions

• What is the selling strategy maximizing Expected Discounted Revenue?

• How to set the parameters of this strategy (e.g. bidding period, fixed sale price) optimally?

• How does the optimal strategy compare with other possible strategies?

• In a given industrial environment, which strategy is more robust?
Talk Outline

1. Analysis
2. Numerical Experiments
3. Conclusion & Managerial Insights
Dynamic Mechanism Design

Bidder i has private valuation

Bidders arrive in sequence

Market opens

Bidder i arrives at $t_i$

Transaction occurs (Allocation, Payment)

Participants are affected by the transaction timing

What is the optimal mechanism (selling time, allocation, payment) in this dynamic environment?
Problem Formulation

Seller with discount factor $\alpha$ opens market at $t = 0$

Bidder $i$ arrives at $t_i$

Transaction occurs at $S$ (endogeneous)

Arrival Process is Renewal

Bidder $i$ has discount factor $\alpha$ and type $\varphi_i = (v_i, t_i)$

What is the mechanism:

$\psi = (\text{Stopping Time } S, \text{ Allocation } q, \text{ Payment } y)$

Maximizing seller’s objective $\mathbb{E}[\alpha^S \sum_i y_i ]$

Subject to:

Each bidder $i$ maximizes $\mathbb{E}[\alpha^{S - t_i} (q_i v_i - y_i) | v_i ]$
Methodology & Assumptions

• Buyers are self-interested ➔ Equilibrium/Game Theoretic Analysis

• Buyers’ private valuations are independent and drawn from the same known distribution

• Model investigates unit demand case

• The seller and buyers discount factors are the same

• All data except individual valuations is public knowledge
Optimal Single Item Mechanism

• Optimal sales strategy derived from this model:

  Set a fixed price $p^*$ such that:

  $$p^* \leq \frac{1 + F + U}{f + u} \quad \text{and} \quad p^* \geq \frac{G(1 + F + W)}{1 + G(1 + F + W)}$$

• $p^*$ depends on the arrival rate and discount factor…
Optimal Multiple Items Mechanism

Optimal sales strategy when selling $K>1$ identical items:

Set an increasing sequence of prices:

$p^{*1} < p^{*2} < p^{*3} < ... < p^{*K}$

- Price for the first sale
- Price for the last sale

Model provides easily implementable formulas for price computation

Temporal incentive compatibility effect
Performance Benchmark

- Online auction model (e.g. eBay):

  Reserve price $p_0$ is set

  Market opens

  Time

  Auction ends at $L$

  Ordering of valuations $v_1 < v_2 < \ldots < v_{N-1} < v_N$

  Revenue $R = \max(v_{N-1}, p_0)$ if $v_N > p_0$

  Expected Discounted Revenue $\alpha^L \mathbb{E}[R]$

- In our experiments, $p_0$ and $L$ are set optimally
Machine-Tool Sale Continued

- In the NC-machine tool sale example, **optimal sales strategy** is
  \[(p^*_1; p^*_2; p^*_3; p^*_4) = (221.2k; 222.3k; 223.7k; 225.5k)\]
  \[\text{EDR} = $866k\]

- The best **single price** strategy is \(p = $222.98k\)
  \[\text{EDR} = $864k\]

- The best **auction** strategy is \(L = 90\text{ days} ; p_0 = $200k\)
  \[\text{EDR} = $841k\]
Discounted Revenue Comparison

Data: Valuations U[0,10], Arrivals Poisson(1)
Mechanism Robustness

Data: Valuations $U[0,10]$, Arrivals Poisson(1)
Data: Valuations U[0,10], Arrivals Poisson(1)
## Experimental Summary

<table>
<thead>
<tr>
<th>Value of Time</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Discounted Revenue</td>
<td>Fixed Price -</td>
<td>Fixed Price ++</td>
</tr>
<tr>
<td>Parameter Robustness</td>
<td>Auction++</td>
<td>Auction -</td>
</tr>
<tr>
<td>Analysis Requirements/Flexibility</td>
<td>Auction+</td>
<td>Fixed Price +</td>
</tr>
</tbody>
</table>

Missing dimension: Number of items sold!
Conclusion

• Mathematical model for setting prices in time-sensitive market environments:

\[ p \geq \frac{1}{f} \frac{F}{\Omega} \quad \text{and} \quad p \geq \frac{G}{1 + G} \frac{F}{\Omega} \]

• In practical implementations, robustness must be considered carefully

**Future work:**
- Multi-unit benchmark
- Dynamic learning/Adaptive Pricing
- …
Impatient Bidders

• The impatient bidders problem is obtained from the previous formulation by adding a constraint

• The optimal mechanism $OFP$ we derived is also feasible for this smaller feasible space

The mechanism $OFP$ is also optimal with impatient bidders!!!
Dynamic Model: Discounted Revenue Comparison

Data: Valuations U[0,10], Arrivals Poisson(1)
Revenue Volatility

Data: Valuations $U[0,10]$, Arrivals Poisson(1)
Mechanism Robustness 1

Data: Valuations U[0,10], Arrivals Poisson(1)
Robustness Test Range: Parameter +/- 5%
Me Mechanism Robustness 2

Data: Valuations $U[0,10]$, Arrivals Poisson(1)
Robustness Test Range: Parameter +/- 5%
Conclusion

• Optimal dynamic sale policy in closed form:

\[ p \left( \frac{1}{f_p} \right) \text{ or } p \left( \frac{G_p F_p}{1 + G_p F_p} \right) \]

• But mechanism choice is a bit more subtle in practice.

Future work:  
- Multi-unit benchmark (B2C)  
- Different time discount factors  
- Dynamic learning/Adaptive pricing
Independent Private Values model: common knowledge is that $v_1, v_2, v_3, \ldots$ are independent and follow the same distribution.
Background: Revelation Principle

• Any market mechanism can be stated as:

\[
\text{Strategy Space} \rightarrow \text{Outcome Space} \quad s \rightarrow [q(s), y(s)]
\]

• Bidders will implicitly use strategy functions:

\[
\text{Type Space} \rightarrow \text{Strategy Space} \rightarrow \text{Outcome Space} \\
\varphi \rightarrow s(\varphi) \rightarrow [q(s(\varphi)), y(s(\varphi))]
\]

• Consider now the following direct mechanism:

\[
\text{Type Space} \rightarrow \text{Outcome Space} \\
\varphi \rightarrow [q(s(\varphi)), y(s(\varphi))]
\]

Without loss of generality, we can restrict our search to Direct Revelation Mechanisms!
Model Formulation

Seller with discount factor $\alpha$ opens market at $t = 0$

Bidder $i$ arrives at $t_i$

Transaction occurs at $S$ (endogeneous)

Arrival Process is Renewal

Bidder $i$ has discount factor $\alpha$ and type $\varphi_i = (v_i, t_i)$

$v_i$ follows an Independent Private Values model with distribution $(f(.), F(.))$

Inter-arrival distribution $x$ has transform $G(z) = E[ z^x ]$
The Mechanism Design Problem

- Define $S \min \left\{ n \nvdash 1 : \bigotimes_{i=1}^{n} q_i^n \bigotimes 1 \right\}$ (stopping time)
  
  $U_i \nvdash v_i \nvdash E \left[ \bigotimes_{i=1}^{n} q_i^S \bigotimes y_i^S \bigotimes v_i \right]$ (utility of bidder $i$)

- Our mechanism design problem can be stated as:

Maximize:  
$U_0 \nvdash E \left[ \bigotimes_{i=1}^{n} y_i^S \bigotimes 1 \right]$

Subject to:  
(IR) \begin{cases} 
U_i \nvdash v_i \nvdash 0 \\
\text{for all } i \nvdash 1 \text{ and } v_i \nvdash V 
\end{cases}

(IC) \begin{cases} 
U_i \nvdash v_i \nvdash E \left[ \bigotimes_{i=1}^{n} q_i^S \bigotimes y_i^S \bigotimes v_i \right] \\
\text{for all } i \nvdash 1 \text{ and } v_i \nvdash V^2 
\end{cases}
Theorem: An optimal solution to the problem:

\[
\text{Maximize } U_0 \prod_s \mathbb{E} \left[ \prod_i s_i y_i^s \right] \\
\text{Subject to (Individual Rationality)} \\
\text{(Incentive Compatibility)}
\]

can be obtained by solving for \( q \) and \( S \) in

\[
\text{Maximize } U_0 \prod_s \mathbb{E} \left[ \prod_i s_i \left( v_i \mathbb{E} \left[ \frac{1_{A_i \cup U}}{f_i U} \right] q_i^s \right) \right] \\
\text{and set } y \text{ such that:}
\]

\[
y_i^s \prod v_i q_i^s \prod v_i E \left[ \prod_i s_i \right], v^s \prod v \rightarrow v
\]
Dynamic Programming Solution

- The maximization of
  \[ U_0 \mathbb{E} \max_{s \in S} \left( \mathbb{E}_i \left( v_i \frac{1_{\Theta_i \in \Theta}}{f_i} \right) q_i \right) \]
  is equivalent to solving the following infinite horizon discounted optimal stopping problem (dynamic program):

  \[
  \begin{cases}
  J \max_{\Theta, \Theta, w} \quad \Theta, \Theta, w \\
  g(\Theta, w) \max_{\Theta, w} \quad \Theta, w \\
  w \not\in v \frac{1_{\Theta \in \Theta}}{f} \\
  \end{cases}
  \]
  \[(\text{Bellmann equation})\]
  \[(\text{state equation})\]
  \[(\text{random noise})\]

- Its one-look ahead stopping set
  \[ S \uparrow x : x \in \Theta, \Theta, w \]
  is optimal!

In the original problem, the optimal mechanism is to sell for a price of \( p^* \) to the first bidder \( S \) such that \( v_S \uparrow p^* \), where \( p^* \) satisfies

\[ p^* \leq \frac{1_{\Theta^* \in \Theta}}{f} \leq p^* \frac{\Theta^* \uparrow \Theta^* \Theta^* \Theta^* \Theta^* \Theta^*}{1_{\Theta^* \in \Theta} \Theta^* \Theta^* \Theta^* \Theta^* \Theta^*} \]
Static Mechanism Benchmark

Data: Valuations U[0,10]