Intertemporal mixed bundling and buying frenzies

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and

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By initially selling goods only in bundles and subsequently selling unsold units individually, a multiproduct seller can create a buying frenzy in which his profit is higher than it would be if he sold all units individually at their market clearing prices. In this frenzy, high-valuation customers buy a bundle because they expect quantity rationing when units are sold individually. Their purchases reduce the quantity to be sold individually, causing the shortages that result in rationing. The bundle’s price exceeds the sum of the individual prices, a fact observed in markets for rock concert tickets.

1. Introduction

During the summer rock concert season, many amphitheaters in the United States sell tickets to selected concerts using a strategy that we call intertemporal mixed bundling. The amphitheaters divide the selling process into two time periods. In the first period, they sell tickets only in bundles called subscription series. In the second, they sell any remaining tickets individually. This selling mechanism can be viewed as a form of what Adams and Yellen (1976) called mixed bundling, where goods are sold both in bundles and individually. However, unlike Adams and Yellen’s version of mixed bundling, in which the bundle price must be less than the sum of the individual good’s prices, almost all of these amphitheaters sell the subscription series at a price that exceeds the sum of the individual ticket prices. This suggests that intertemporal mixed bundling is used for reasons that are different from those identified by Adams and Yellen.

We present a theory of intertemporal mixed bundling that builds on ideas in the buying frenzy literature to explain this pricing behavior. We show that a capacity-constrained seller, by initially offering goods for sale only in a bundle and subsequently

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sells unsold units individually, can create a buying frenzy. In this frenzy, high-valuation customers, expecting that there will be excess demand and quantity rationing when goods are sold individually, choose to purchase a bundle to avoid the rationing. These high-valuation customers purchase more units than they would have in a market-clearing equilibrium. These “additional purchases” cause the shortage that results in rationing when goods are sold individually, fulfilling expectations.

We consider a capacity-constrained monopolistic seller who sells two goods, A and B. He faces four types of customers: high-A types who value one unit of A above its market-clearing price if units were sold individually, and one unit of B below its market-clearing price; high-B types who value one unit of B above its market-clearing price, and one unit of A below its market clearing price; low-A types who value one unit of good A at its market-clearing price; and low-B types who value a unit of B at its market-clearing price.

The seller offers for sale, in period 1, one unit of A and one unit of B only in a bundle. In period 2 he sells any unsold units individually. The market exhibits frenzy purchasing when high-A types buy a bundle to guarantee that they obtain A rather than wait to buy A individually when they believe rationing may occur. As a result, these customers purchase a unit of B that they would not have purchased had the goods only been sold individually. This reduces the available quantity of B, creating a situation where customers who wait to purchase B individually will be rationed. Knowing this, high-B types purchase a bundle to avoid rationing. This reduces the quantity of A available, which means that customers who wait to purchase A will be rationed, fulfilling the expectations that rationing will occur. That is, once high-valuation customers purchase a bundle, fewer units remain to be purchased individually (at the period 2 price) than there are low-valuation customers demanding individual units. Thus customers who purchase units individually are quantity rationed.

In our model, units sold individually in period 2 of the buying frenzy equilibrium are sold at the same price that would clear the market if all units were sold individually. So using intertemporal mixed bundling will be more profitable than selling goods individually when the bundle’s price exceeds the sum of the component’s market-clearing prices.

Since a high-A type values a unit of B below the market-clearing price, bundling imposes an opportunity cost on the seller relative to selling goods individually. He implicitly sells a unit of B in a bundle purchased by a high-A type for a price below the market-clearing price. Bundling also confers a benefit on the seller. The high-A type implicitly pays a premium for A in excess of the market-clearing price in exchange for avoiding the possibility of being rationed. Bundling is profitable if the premium exceeds the opportunity cost.

1 On high-demand weekends, hotels often initially require a two-night minimum. During the weekend they offer unsold roomnights individually. To apply our theory, some customers must value one night (say Friday) more highly while others value the other night (i.e., Saturday) more. An alternate explanation suggests that for fairness reasons hotels cannot raise rates but can require the purchase of multiple nights at the existing rate.

2 This explanation assumes no resale of the goods. We discuss this assumption in the context of our model later in the article.

3 At Camden Yards, Baltimore Orioles season tickets (and smaller bundles) are sold initially, then unsold tickets are sold individually. In past seasons virtually all home games sold out and virtually all good seats were purchased in a bundle in advance.

4 We show in Section 6 that intertemporal mixed bundling can also be profitable when the bundle price is less than the sum of the individual component market-clearing prices if it increases the number of tickets sold relative to the number sold in the market-clearing equilibrium.

5 There is of course a symmetric argument with respect to selling bundles to customers that value B highly.
This explanation suggests at least two conditions that are necessary for intertemporal mixed bundling to be profitable. First, each high-valuation customer must expect that other high-valuation customers will purchase the bundle in period 1. Since the purchase of bundles results in a shortage and ultimately rationing in period 2, an individual who expects others to purchase a bundle finds it rational in equilibrium to also purchase the bundle. This is frenzy purchasing behavior.

Second, a high-\(A\) type must have a valuation for good \(B\) that is below but close to \(B\)’s individual good market-clearing price (and, similarly, the high-\(B\) type’s valuation for good \(A\) must be close to \(A\)’s market-clearing price). This implies a small opportunity cost of including a unit of \(B\) in a bundle purchased by a high-\(A\) type.

Intertemporal mixed bundling can be viewed as a self-selection mechanism in which the seller separates high-valuation customers from low-valuation customers by offering a menu of purchasing options. We shall compare this self-selection mechanism with an optimal direct-report mechanism in which customers announce their valuation type and the seller sells them a (possibly fractional) quantity of each good at a specified price.\(^6\) The mechanism that maximizes the seller’s profits is identical to intertemporal mixed bundling when the distribution of customers is symmetric across goods. When the distribution is asymmetric, bundling cannot duplicate the optimal mechanism, since it affords the seller fewer instruments than there are customer types. We argue that we observe intertemporal mixed bundling rather than the more complex optimal mechanism because it would be impractical to implement the optimal mechanism.

Although our results and those of Adams and Yellen can both be viewed as self-selection mechanisms, there are many differences between them. As already noted, in our model the bundle is sold for a price that exceeds the sum of the individual prices, whereas this is not possible under the static formulation. In addition, under static mixed bundling, units are sold individually to extract surplus from high-valuation customers, whereas bundles are sold to customers with lower valuations. In our model the bundle price is used to extract surplus from high-value customers and individual components are sold to the lower-value customers.

Our results actually bear a greater resemblance to results found in the priority pricing literature.\(^7\) In that literature, a seller who faces exogenous randomness in a market parameter offers customers the option of paying a high price to obtain the good with certainty or a lower price at which the customer faces a positive probability of rationing. In our model the possibility of rationing is generated endogenously through bundling and the resulting frenzy. Thus the primary role of bundling is to provide a mechanism whereby the seller can credibly quantity ration customers who attempt to purchase a unit individually at what would be the market-clearing price.

The article is organized as follows. Section 2 reviews the literature. In Section 3 we present a numerical example using one good to provide intuition underlying the use of intertemporal bundling. In Section 4 we show that intertemporal mixed bundling can result in an equilibrium of a price-setting game with more than one type of good. Section 5 compares intertemporal mixed bundling to an optimal direct-report mechanism. In Section 6 we extend the model to encompass several stylized facts regarding the summer concert market. Concluding remarks are in Section 7.

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\(^6\) We thank an anonymous referee for pointing out this interpretation and inspiring all subsequent discussion of mechanism design.

\(^7\) See Tschirhart and Jen (1979). In our explanation, the seller can only sell at the premium price by bundling. With priority pricing, the seller can set such a price when selling an individual good. Also, bundling is only useful when it induces a buying frenzy (see DeGraba, 1995). No frenzy is needed with priority pricing.
2. Literature review

Stigler (1968) demonstrated that a multiproduct monopolist facing customers with negatively correlated demand could earn higher profits by selling goods in a bundle instead of individually. Adams and Yellen (1976) showed that a seller could increase profits further by selling the goods both in bundles and individually. Schmалensee (1982) showed that a single-product monopolist that bundles its monopoly product with a competitively produced good may be able to price discriminate using mixed bundling if there is a negative correlation between buyers’ reservation prices for the bundled goods.

This article shows that mixed bundling in an intertemporal setting can be used for a different reason. Specifically, it can create a buying frenzy in which customers pay a premium to avoid being rationed. In doing so, our explanation extends ideas from the optimal auction, priority pricing, and buying frenzy literatures.

The optimal auction literature (for example, Bulow and Roberts (1989)) shows that if a monopolist faces a demand curve with a kink so that the marginal revenue curve is nonmonotonic, the profit-maximizing sales strategy is to sell units above the kink at a high price with certainty and sell units around the kink at a lower price via lottery. While our explanation also involves some customers purchasing at a high price to avoid purchasing via a lottery, it is not necessary for the marginal revenue function to be nonmonotonic for intertemporal mixed bundling to be effective.

The priority pricing literature, including works by Harris and Raviv (1981), Tschirhart and Jen (1979), and DeGraba and O’Hara (1992), shows that when customers face the prospect of quantity rationing at a given price, they will pay a premium above that price to guarantee delivery. Bulow and Klemperer (1994) extend this idea, focusing on how the actions of one customer can affect the decisions of other customers. They show that one customer’s decision to buy increases the incentive for others to buy, creating a “feedback effect” that can result in frenzy purchasing on the part of customers in a perfectly competitive market. DeGraba (1995) shows that a monopolist can manipulate such feedback effects by choosing production levels that create conditions of excess demand resulting in frenzy buying. In such a frenzy all customers pay a premium to ensure delivery even though all customers would be better off if none attempted to purchase at the premium price.

There are other explanations for rationing in the literature. One strand of the rationing literature (see, for example, Gilbert and Klemperer (forthcoming)) treats rationing as a “reward for incurring a sunk cost.” The idea here is that a buyer must sink an investment to use a good before some randomness affecting the price/availability of that good is realized. If the price for that good always clears the market, then the expected return from sinking the investment will be low, so no one sinks the investment. On the other hand, if the market price is less than the market-clearing price in some states of the world and the good is rationed, the payoff from the sunk cost increases because the market price extracts less of the rents in these states. This makes the investment in the sunk cost profitable. Clearly our story is different from this, since no customer sinks any costs in our model in order to buy the good.

In a second strand, Basu (1987) and Becker (1991) generate equilibrium excess demand in a market where the utility of one customer for a good is an increasing function of the number of other customers who demand the good. In our model the valuation a customer places on a good is independent of who else purchases the good.

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8 McAfee, McMillan, and Whinston (1989) and Schmalensee (1984) showed that bundling can be an optimal strategy even when reservation prices are not negatively correlated.
A third strand of the literature assumes that demand for the good in the future depends on the price of the good today. (See Allen and Faulhaber (1991), Slade (1991), and Haddock and McChesney (1994).) In such cases the seller may set today's price below the market-clearing level to increase the demand in the future. Since customers in our model are only buying the good once, such considerations are not relevant.

3. Numerical example with one good

There is a monopolistic seller who sells a good for which a capacity constraint allows him to produce an output of mass 1. There are two continua of customer types—high and low—that might purchase the good. Each high type values the first unit at $2 and the second unit at $(1 - \epsilon)$, for \( \epsilon \) positive but close to zero. Each low type values the first unit at $1 and has no value for a subsequent unit. The high types have mass \( \frac{1}{3} \), and the low types have mass \( \frac{2}{3} \). The seller cannot distinguish between types.

The good is sold in two periods. In period 1 the seller announces a period-1 price for the good and also commits to a period-2 price. In this setting, the seller can earn more by selling only bundles of two units in period 1 and single units in period 2 than he can by selling all units individually at the market-clearing price. Given the seller's informational constraints, this method of selling maximizes his profits.\(^9\)

Suppose the seller sets a bundled price in period 1 of $(2\frac{1}{2} - \epsilon)$ and commits to a period-2 price of $1 for a unit, and further suppose that all high types choose to purchase a bundle at the period-1 bundled price. In such a case, a total of \( \frac{2}{3} \) units will be purchased by the high types in period 1. This leaves \( \frac{1}{3} \) units available in period 2 for the low types, of mass \( \frac{2}{3} \) to purchase. Thus, in period 2 a low type will be quantity rationed with probability \( \frac{1}{2} \).

A high type who purchases a bundle in period 1 receives a surplus of \( \frac{1}{2} \) (calculated as \( 3 - \epsilon - (2\frac{1}{2} - \epsilon) \)). If a high type decided to purchase in period 2, he would also receive an expected surplus of \( \frac{1}{2} \), purchasing a unit in period 2 with probability \( \frac{1}{2} \) at a price of $1. If all high types purchase a bundle in period 1, no individual high type can make himself better off by trying to purchase in period 2. Thus it is individually rational for all high types to buy a bundle in period 1.

The profit received by the seller from this pricing strategy is \( $(\frac{7}{6} - (\frac{1}{2})\epsilon)$ (calculated as \( \frac{1}{2}(2\frac{1}{2} - \epsilon) + \frac{1}{2} \)). Had the seller sold all units at the market-clearing price of $1 he would have earned a profit of $1. $(\frac{7}{6} - (\frac{1}{2})\epsilon)$ is greater than $1 if \( \epsilon < \frac{1}{2} \). Thus for \( \epsilon < \frac{1}{2} \), the seller earns a higher payoff by selling only bundles in the first period and individual units in the second period than he earns by selling all units individually at the market-clearing price.

The equilibrium sketched above provides two observations that will be helpful in understanding the role played by intertemporal mixed bundling in the two-good model. First, the equilibrium described is similar to the frenzy equilibrium described in DeGraba (1995). This means that in equilibrium, a high-type customer finds it optimal to purchase a bundle in period 1 only because all other high types also purchase in period 1. At the prices given, if no high type purchased a bundle in period 1, each could buy a unit with certainty in period 2 and obtain a surplus of 1.\(^{10}\)

\(^9\) In the Appendix we show that intertemporal mixed bundling implements the solution to the optimal direct-report mechanism. This solution is identical to the solution of the symmetric version of the two-good model in Section 4.

\(^{10}\) Note that in DeGraba (1995), customers purchase the good before they know its true value to them. Some customers therefore pay a price that exceeds their ex post valuation for the good. In this article no customer pays a price that exceeds his valuation of the bundle he purchases.
Second, bundling will be profitable when the units that are purchased in a bundle that would not be purchased in a market-clearing regime are valued by the bundle purchaser “sufficiently close to” the market-clearing price. Under bundling, each high type purchases a unit that he values \( e \) less than the market-clearing price. He would not have purchased this unit had bundling not been used. Thus the seller incurs an opportunity cost of \( e \) for each such unit he sells relative to what he would have received had he sold all units at the market-clearing price. The seller also receives a benefit from bundling, in that the high types pay a premium to avoid the possibility of being rationed in period 2. When \( e < \frac{1}{2} \), the premium the seller receives from bundling exceeds the opportunity cost incurred from bundling.

4. Equilibrium analysis with two goods

A multiproduct monopolist (the seller) produces two goods, \( A \) and \( B \) (for simplicity), at zero marginal cost. The seller is capacity constrained and can produce and sell at most \( K \) units of each good.\(^{11} \) Once a unit is purchased it cannot be resold.\(^{12} \)

There are four valuations a customer can have for a good, \( v_4 > v_3 > v_2 > v_1 \). While this could give rise to 16 combinations of valuations and therefore 16 different customer types, we will consider only four types of risk-neutral customers: high-\( A \) (\( H_A \)) with values for \( A \) and \( B \) of \((v_4, v_2)\) respectively; high-\( B \) (\( H_B \)) with values for \( A \) and \( B \) of \((v_2, v_4)\); low-\( A \) (\( L_A \)) with values \((v_3, v_1)\); and low-\( B \) (\( L_B \)) with values \((v_1, v_3)\). This configuration is consistent with there being two broad categories of customer, \( A \) and \( B \), with high- and low-valuation customers within each category. Every \( A \) will pay more for a unit of \( A \) than any \( B \), and every \( B \) will pay more for a unit of \( B \) than any \( A \). Also, positive correlation in valuation across goods within each category implies that an \( H_A \) type pays more for a unit of \( B \) than an \( L_A \) type pays, and similarly an \( H_B \) type pays more for a unit of \( A \) than an \( L_B \) type pays.\(^{13} \)

The value a customer places on one good is independent of whether or not he purchases the other good. Each customer values only one unit of each good. There is a continuum of each type of customer. Each continuum of \( H \) types has mass of \( n_{Hi} \), and each continuum of \( L \) types has mass \( n_{Li} \) for \( i \in \{A, B\} \). “Not \( i \)” is denoted \( \overline{i} \).

Each good is sold in two periods. The seller announces a period-1 price. Customers observe this price and decide whether to purchase in period 1 or wait until period 2. The seller observes the quantity purchased in period 1 and then sets a period-2 price.\(^{14} \) Customers observe this price and then make their period-2 purchase decisions (see Figure 1). In any period, if the quantity demanded exceeds the available capacity, then customers are rationed randomly.

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\(^{11} \) The seller cannot produce \( K + x \) units of \( A \) and \( K - x \) units of \( B \). This assumption is quite natural for concerts, where venue size is fixed over time and thus the number of seats available for one concert is equal to the number of seats available for any other concert.

\(^{12} \) Resale markets for concert tickets exist but their illegality limits participation, resulting in some nonprice rationing. Since nonprice rationing drives our results, they may apply when resale is imperfect as well as completely impossible. Interestingly, in Los Angeles, where there are well-developed resale markets, no major amphitheaters use subscription series.

\(^{13} \) It might be helpful to think of \( H \) types as rich customers and \( L \) types as poor customers such that for any good, the rich \( A \)-type customer will pay more than a poor \( A \)-type customer, similarly for \( B \) types.

\(^{14} \) Our results hold if the seller commits to a period-2 price when announcing the period-1 price. Prices are set sequentially to show that if the seller’s pricing induces excess demand in period 2, commitment is not necessary. Concert promoters do commit to period-2 prices by publishing each ticket’s price when announcing subscription rates.
TABLE 1  Summary of Customer Types

<table>
<thead>
<tr>
<th>Type of Customer</th>
<th>Mass of Customers</th>
<th>Reservation Prices V_A V_B V_A + V_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_A</td>
<td>n_{H_A}</td>
<td>v_4 v_2 v_4 + v_2</td>
</tr>
<tr>
<td>L_A</td>
<td>n_{L_A}</td>
<td>v_3 v_1 v_3 + v_1</td>
</tr>
<tr>
<td>H_B</td>
<td>n_{H_B}</td>
<td>v_2 v_4 v_4 + v_2</td>
</tr>
<tr>
<td>L_B</td>
<td>n_{L_B}</td>
<td>v_2 v_3 v_3 + v_1</td>
</tr>
</tbody>
</table>

We impose the following restrictions:

\[ K v_3 > n_{H_A} v_4 \]  \hspace{1cm} (1a)
\[ K v_3 > n_{H_B} v_4 \]  \hspace{1cm} (1b)
\[ n_{H_i} + n_{L_i} = K \quad \text{for } i = A, B. \]  \hspace{1cm} (2)

Equation (1) says that if the seller were to sell the goods individually, he would earn a higher payoff by setting a price of \( v_3 \) and selling his capacity than he would by setting a price of \( v_4 \) and selling to only high-type customers. Equation (2) says that for each good \( i \), the mass of \( i \)-type customers is equal to the seller’s capacity. We impose (2) to ensure (along with (1)) that there is a price at which each market will clear exactly if goods are sold individually.

In this setting we compare two possible selling strategies: single-good pricing and intertemporal mixed bundling.\(^{15}\) Under single-good pricing the seller can announce a price for each individual good in period 1. After observing period-1 purchases the seller sets period-2 prices. Under intertemporal mixed bundling the seller sets only a price for the bundle containing one unit of \( A \) and one unit of \( B \) in period 1. After observing period-1 sales the seller can set a price for each good in period 2.

Given this structure, we construct the following game played by the seller and the customers. The seller chooses \( \{ P_{A1}, P_{B1}, P_A, P_{A2}, P_{B2} \} \), where \( P_{A1} \) and \( P_{B1} \) are the period-1 individual prices for \( A \) and \( B \) if single-good pricing is chosen, \( P_b \) is the period-1 bundle price if mixed bundling is chosen,\(^{16}\) and \( P_{A2} \) and \( P_{B2} \) are functions that map each realization of the game in period 1 into period-2 prices.

The strategy for each customer is to choose the set \( \{ R_{A1t}, R_{B1t}, R_{A2t}, R_{B2t} \} \), where \( R_{ijt} \) is the reservation price for good \( i \) in period \( j \) set by customer type \( t \in \{ H_A, H_B, L_A, L_B \} \) and \( R_{bt} \) is the reservation price of a type-\( t \) customer for the bundle. The seller’s payoff is his total sales revenues. Each customer’s (expected) payoff is the surplus realized from his buying strategy. Only subgame-perfect Nash equilibria are considered.

The results of the previous section suggest that an intertemporal mixed-bundling equilibrium will occur when the premium the seller can extract for the bundle exceeds the opportunity cost from selling some units below the single-good market-clearing\(^{15}\) We could also consider the seller selling bundles in both periods and selling the goods individually in period 1 and in a bundle in period 2. Bundling in period 1 and selling separately in period 2 dominates both of these strategies. This fact is a direct result of the analysis of the optimal solution in Section 5.

\(^{16}\) Formally, if the seller chooses to bundle in period 1 we can think of \( P_{A1} \) and \( P_{B1} \) as being set equal to infinity.

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prices. This observation gives rise to the following general condition under which an intertemporal mixed-bundling equilibrium exists.

**Proposition 1.** Assume without loss of generality that \( n_{LA} \leq n_{LB} \).

If \( (v_4 - v_3)[1 - (K - (n_{HA} + n_{HB}))/n_{LA}] > v_3 - v_2 \), then there exists an intertemporal mixed-bundling equilibrium, in which the bundle price is

\[
v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))/n_{LA},
\]

which exceeds the sum of the individual-good prices. The seller earns a higher payoff in this frenzy equilibrium than if he sold all units at the market-clearing prices.

**Proof.** See the Appendix.

The condition simply says that when the additional revenue a seller earns by selling a unit of \( A \) in a bundle instead of in the market-clearing equilibrium exceeds the opportunity cost of selling \( B \) in that bundle, a frenzy equilibrium exists. The left-hand side represents the additional revenue. An \( H_A \) type obtains the expected surplus of purchasing a unit of \( A \) in period 2, which is \( (v_4 - v_3)(K - (n_{HA} + n_{HB}))/n_{LA} \). The seller captures the rest of the surplus, which is \( v_4 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))/n_{LA} \). Subtracting \( v_3 \), the price the seller would have gotten for the unit of \( A \) in the market-clearing equilibrium, yields the additional revenue from selling the unit in the bundle. The right-hand side, \( v_3 - v_2 \), is the net opportunity cost of selling a unit of \( B \) via bundling to an \( H_A \) type who values it at \( v_2 \) rather than selling it at the market-clearing price of \( v_3 \).

An alternative interpretation can be obtained by thinking of intertemporal bundling as a self-selection mechanism in which \( H_i \) customers weakly prefer purchasing a bundle to purchasing a unit of \( i \) individually. Rewrite Proposition 1’s condition as \( (v_4 + v_2) - (v_4 - v_3)(K - (n_{HA} + n_{HB}))/n_{LA} > 2v_3 \). The two terms on the left-hand side represent the maximum surplus that can be extracted from an \( H_A \) type and still have him prefer to purchase a bundle rather than wait to purchase a unit of \( A \) in period 2 at the price \( v_3 \) (i.e., the incentive-compatibility constraint). The first term is the value of the bundle to an \( H \) type, and the second term is the expected surplus an \( H_A \) would receive by waiting to purchase a unit of \( A \) in period 2. Thus the left-hand side is the price of the bundle. The right-hand side is the revenue the seller would have earned had he sold the two units individually at the market-clearing price. Thus this condition says that when the revenue from selling a bundle exceeds the revenue that could be obtained by selling two units individually, a frenzy equilibrium exists.

The seller extracts from each \( H \) type the surplus of the \( H_i \) type with the lowest probability of being rationed (in this case \( H_A \) types, since \( n_{LA} \leq n_{LB} \)). Hence the price of the bundle makes the \( H_A \) customers indifferent between buying the bundle or waiting until period 2 to purchase a unit of \( A \) individually, given that he believes all other \( H \) types will purchase a bundle in period 1. \( H_B \) types, however, strictly prefer to purchase the bundle in period 1 as opposed to waiting, since an \( H_B \) type who waits until period 2
has a higher chance of being rationed than an $H_A$ type.\footnote{That is, an $H_A$ type that deviates must purchase a unit in period 2 when there are $K - (n_{LA} + n_{LB})$ units available and $n_{LB}$ low types trying to buy, while an $H_A$ type would buy in a market with the same number of units available but only $n_{LA}$ low types trying to buy.} If the seller tried to increase the bundle price to capture the additional surplus from $H_B$-type customers, he would price the $H_A$-type customers out of the period-1 market, which would destroy the buying frenzy. This will be important when comparing this result to the optimal direct-report mechanism.

The two observations made in Section 3 can help explain our results. First, in this equilibrium, the $H$ types exhibit the same frenzy behavior as the high types in Section 3. That is, it is individually rational for each $H$ type to purchase the bundle at a premium only when he believes all other $H$ types will purchase the bundle for the same premium. In equilibrium these expectations are fulfilled. (This is reflected in the term $(K - (n_{HA} + n_{HB}))/n_{LA}$. The $n_{HB}$ indicates that because of bundling, $n_{HB}$ units of $A$ are not available to be purchased in period 2, which causes the excess demand.) Note that all $H$ types would be strictly better off if none of them purchased the bundle in period 1. If no $H$ type purchased a bundle in period 1, then all $H$ types could purchase a unit in period 2 with certainty at the market-clearing price of $v_3$.\footnote{Quantity rationing occurs because high-value customers impose an externality on the market. $H_A$ types impose an externality on $H_B$ types because by purchasing a bundle, they purchase a unit of $B$ that they otherwise would not purchase. $H_B$ types impose a similar externality on $H_A$ types.}

This explanation suggests (correctly) that there is also an equilibrium in which there is no intertemporal mixed bundling.\footnote{In fact there is a continuum of frenzy equilibria along with one individual-good market-clearing price equilibrium. Each equilibrium in the continuum has the same structure, which is that each $H$ type chooses the same reservation price for the bundle, and the seller sets that price.} Why then should we focus on the mixed-bundling equilibrium in which there is excess demand as the appropriate equilibrium when there exists a market-clearing equilibrium?\footnote{We thank a referee for noting that the seller could make the frenzy equilibrium unique by limiting the quantity of tickets sold individually. Promoters impose no such limitations, suggesting that such commitments may be difficult to make credibly, or that customers may not process such detailed information precisely enough to affect behavior.}

First, we have already mentioned in the Introduction that in fact we observe bundles being sold at prices that exceed the sum of the individual prices, an observation for which there is no current existing explanation in the literature. Second, Mohammed (1999) presents empirical evidence that is consistent with our theory.\footnote{He shows that sales of all acts increase as a result of being in subscription series. No direct data on sellouts exist because amphitheaters have a pavilion section and a lawn section but only report total ticket sales. Our theory best applies to pavilion seat sellouts. Anecdotes indicate that pavilion seats are all sold to purchasers of subscription series.}

Aside from empirical evidence, one might also look at institutional factors to see if they suggest reasons why customers might focus on one or another type of equilibrium. For example, our model has considered a market in which there is an equilibrium at which the market clears exactly. Yet it is well known (and still a source of interest to economists) that many individual concerts experience excess demand and quantity rationing. Thus if customers in venues where intertemporal mixed bundling is used observe that concerts at venues where tickets are sold individually experience rationing, they should correctly expect that there would also be rationing in period 2 where intertemporal mixed bundling is used. Thus, high-valuation customers would naturally focus on purchasing a bundle in period 1 to avoid this rationing and, knowing that all other high types would likely focus on purchasing a bundle, would be willing to pay
a premium for the bundle that reflected not only the level of excess demand experienced at other venues, but also the additional excess demand resulting from a frenzy.\footnote{This is similar to explanations of bank runs. Depositors, seeing that one bank fails, withdraw their money from their own bank, causing it to fail. Bank runs are rare, due in part to free deposit insurance. We have ruled out second-hand markets, which would provide “excess demand insurance” to high types.}

The second observation from Section 3 was that if the $H$ types have a valuation for the unit that they would not buy at the individual market-clearing price that is “sufficiently close to” the market-clearing price, then bundling will be profitable for the seller. In the model, this observation is borne out by the fact that the condition is most easily met when $v_3 - v_2$ (the right-hand side of the expression) is close to zero. Recalling that $v_3$ would be the market-clearing price and noting that an $H_i$ type would not buy a unit of $-i$ at this price yields the result.

There are two other results regarding the parameter values that tend to make bundling profitable. First, when the left-hand side of the condition is large, then bundling is more profitable. Thus, holding all other parameters fixed and maintaining (1) and (2), the larger the difference between $v_4$ and $v_3$, the more profitable bundling will be. Intertemporal mixed bundling is profitable because it allows the seller to capture a percentage (equal to $1 - (K - (n_{HA} + n_{HB})/n_{LA})$ of the surplus high-type customers would enjoy if the seller had sold all units at the market-clearing price. So the larger this surplus is, the more profitable bundling will be.

Second, increasing the proportion of $H_i$ types (again subject to (1) and (2)) increases the profitability of bundling. The probability of an $H_A$ type being rationed in period 2 is $1 - (K - (n_{HA} + n_{HB})/n_{LA})$, which is decreasing in $n_{HA}$ and $n_{HB}$. Since an $H$ type is willing to pay more for a bundle in period 1 the greater the probability of being rationed in period 2, increasing $n_{HA}$ or $n_{HB}$ increases the profitability of bundling.

Each of these characteristics can be seen by simplifying the condition of the proposition using (2). This results in the expression $n_{HB}(v_4 - v_1) > n_{LA}(v_3 - v_2)$, which easily demonstrates the relationship among the important parameters needed for a frenzy equilibrium to exist. We point out that if we let $n_{Hi} = \frac{1}{3}$ and $n_{Li} = \frac{2}{3}$ for $i \in \{A, B\}$ and set $v_4 = 2$, $v_3 = 1$, and $v_2 = 1 - e$, intertemporal mixed bundling implements the same allocation found in the example in Section 3.

When the seller is restricted to offering only bundles of integer quantities in a two-period framework, the intertemporal mixed-bundling equilibrium maximizes the seller’s payoff.\footnote{It is a simple matter to compare the payoff from the intertemporal mixed-bundling equilibrium to the payoff from any other possible combination of bundled offerings.} In the next section we look at the optimal strategy when fractional allocations of the goods are possible.

5. Optimal direct-report mechanism

We now discuss the relationship between the intertemporal mixed-bundling allocation and the optimal direct-report mechanism allocation. When the distribution of customers is symmetric, intertemporal mixed bundling implements the optimal mechanism. When the distribution is asymmetric, it does not, because intertemporal mixed bundling allows fewer contracts than there are customer types from which to extract rents. That is, there are four customer types but only three possible contracts. With symmetric distributions there are essentially only two types of customers, high and low, and the three contracts provide enough instruments to extract all of the rents that can be extracted given the information constraints.

We begin by characterizing the optimal mechanism and then show why intertemporal mixed bundling only replicates it in the symmetric case. We then suggest that
amphitheaters may implement the intertemporal mixed bundling instead of the more complex optimal mechanism, for transactions cost reasons.

We view the seller’s problem as one of setting up a direct-report game. There are four types of customers among which the seller is unable to distinguish. These types are the same as in Section 4. We add an additional restriction on $v_i$.

$$v_2 n_{HB} > Kv_1 \quad (3a)$$

$$v_2 n_{HA} > Kv_1. \quad (3b)$$

This implies that demand for $A$ and $B$ on the lowest part of the demand curve is inelastic.

Each customer announces a type and receives the contract designated for that type. A contract consists of an allocation of each good and a payment for the allocations. Let $H_i$ be the allocation of good $j \in \{A, B\}$ for those announcing type $H_i$, and $L_i$ be the allocation of good $i$ to those announcing type $L_i$ for $i \in \{A, B\}$. We use lowercase $a$ and $b$ for allocations (i.e., $H_{ab}$) to help distinguish allocations from other values.

The Appendix shows that $L_i^b$ must be zero, so we omit this possibility to simplify the notation. $P_{Hi}$ is the price of the $H_i$ contract and $P_{Li}$ is the price of the $L_i$ contract.

**Proposition 2.** Assume without loss of generality that $n_{LA} \leq n_{LB}$. The seller’s optimal set of contracts involves allocating exactly one unit of each good to customers announcing $H_B$, allocating exactly one unit of $A$ and $1 - D^*$ units of $B$ to those announcing $H_A$, allocating $L^a_{ab}$ units of $A$ to the customers announcing type $L_A$, and allocating $L^b_{ab} + D^*$ units of $B$ to the customers announcing type $L_B$, where

$$D^* = \frac{(v_4 - v_2)(L^a_{ab} - L^b_{ab})}{[(v_4 - v_2)n_{LA} + (v_4 - v_2)n_{HA}]},$$

$$L^a_{aa} = (K - n_{HA} - n_{HB})n_{LA}, \quad \text{and} \quad L^b_{bb} = (K - n_{HA} - n_{HB})n_{LB}.$$

**Proof.** Without loss of generality we consider a direct-revelation mechanism. The following participation and incentive-compatibility constraints must be met.

$$P_{Li} = v_3 L_{ii} \quad (4)$$

$$P_{Hi} \leq v_4 H_{ii} + v_2 H_{i-i} - (v_4 - v_2)L_{ii} \quad (5)$$

$$v_4 H_{ii} + v_2 H_{i-i} - P_{Hi} \geq v_4 H_{ii} + v_2 H_{i-i} - P_{H-i}. \quad (6)$$

Condition (4) is the incentive-compatibility constraint for the $L$ types, and it implies they earn zero surplus. Condition (5) is the incentive-compatibility constraint that says an $H_i$ type announcing $H_i$ must earn as much surplus as he could by announcing $L_i$. Equation (6) is the incentive-compatibility constraint that says an $H_i$ type announcing $H_i$ must earn as much surplus as he could by announcing $H_{-i}$. The constraint that prevents an $H_i$ type from announcing $L_{-i}$ is satisfied because an $H_i$ values a unit of $-i$ at $v_2$ but its price is $v_3$.

Plugging (5) into the seller’s profit function yields

$$\Pi = n_{HA}(v_4 H_{aa} + v_2 H_{ab} - (v_4 - v_2)L_{aa}) + n_{HB}(v_4 H_{ba} + v_2 H_{ba} - (v_4 - v_2)L_{bb})$$

$$+ n_{LA}L_{aa}v_3 + n_{LB}L_{bb}v_3,$$

subject to the capacity constraints that
\[ L_{aa} \leq -(n_{HB} / n_{LA}) H_{aa} - (n_{HA} / n_{LA}) H_{ba} + \frac{K}{n_{LA}} \]  

and

\[ L_{bb} \leq -(n_{HB} / n_{LB}) H_{bb} - (n_{HA} / n_{LB}) H_{ab} + \frac{K}{n_{LB}} \]

and (6), where (7) and (8) are just the constraints that the sum of allocations of good i can’t exceed the seller’s capacity to produce i.

The optimal allocation is the solution to an extensive linear programming problem. We show in the Appendix that \( H_{aa} = 1 \), \( H_{ab} = 1 \), and \( H_{ba} = 1 \) satisfy the first-order conditions of the program, assuming \( H_{ab} = 1 \).

If we were to find that \( H_{ab} = 1 \), then the allocation of intertemporal mixed bundling would be optimal. However, for \( n_{LA} < n_{LB} \) this is not the optimal solution. At this allocation the \( H_A \) and \( H_B \) types pay the same price for the same bundle, but an \( H_B \) that deviated by announcing \( L_B \) would obtain a lower allocation of \( B \) than an \( H_A \) who deviated by announcing \( L_A \) would receive of \( A \). If \( H_A \) types are just indifferent between announcing \( H_A \) and \( L_A \), then an \( H_B \) type must strictly prefer announcing \( H_B \) to \( L_B \). In other words, the constraint (5) would not be binding on the \( H_B \) types.

Thus the seller could raise \( P_{HB} \) without giving \( H_B \) types an incentive to announce \( L_B \). However, simply raising the price of the bundle to an \( H_B \) type when \( H_{ab} = 1 \) would cause him to announce \( H_A \). To raise the bundle price to \( H_B \) types profitably, the seller must also reduce the allocation of \( B \) that those announcing \( H_A \) receive and the payment for the bundle.

Starting with the intertemporal mixed-bundling allocation outlined above (i.e., \( H_{ab} = H_{ba} = 1 \)), reallocating \( D \) units of \( B \) from the bundle designated for those announcing \( H_A \) to the allocation for those announcing \( L_B \) (i.e., lowering \( H_{ab} \) and increasing \( L_{ba} \)) changes the seller’s revenue in three ways. First, it increases his revenue from the \( L_B \) types by \( Dv_3 \), because it increases the quantity sold to these customers. Second, it decreases revenue from \( H_A \) customers by \( Dv_2 \), because it reduces the allocation of \( B \) to a customer announcing \( H_A \) by \( Dn_{HA} \) and therefore the price at which \( H_A \) types are indifferent between announcing \( H_A \) and \( L_A \). Third, it increases the revenue from the \( H_B \) customers by \( n_{HB}(v_4 - v_2)Dn_{HA} \). This happens because reducing \( H_{ab} \) lowers the value of the \( H_A \) bundle to \( H_B \) customers by \( v_3Dn_{HA} \) and reducing the price of the \( H_B \) bundle raises the value to the \( H_B \) customers by \( v_2Dn_{HA} \). Thus the overall change in the seller’s revenue by such a reallocation is \( D(v_3 - v_2) + n_{HB}(v_4 - v_2)Dn_{HA} \). This is clearly positive for all levels of \( D \).

The optimal level of \( D \) is the \( D^* \) for which an \( H_B \) type is indifferent between announcing \( H_A \) and announcing \( L_B \), assuming that \( P_{HB} \) is always set to make the \( H_B \) customers indifferent between announcing \( H_B \) and \( H_A \). For any level of \( D \), the price that equates the surplus to an \( H_B \) type from truthfully announcing \( H_B \) and falsely announcing \( H_A \) satisfies

\[ v_4 + v_2 - P_{HB} = v_4(1 - Dn_{HA}) + v_2 - [(v_4 + v_2(1 - Dn_{HA}) - (v_4 - v_2)L_{aa}^u)], \]

which implies

\[ P_{HB} = v_4(1 + Dn_{HA} - L_{aa}^u) + v_3L_{aa}^u + v_2(1 - Dn_{HA}). \]

Given this pricing rule for \( P_{HB} \), the optimal level of \( D \), \( D^* \) equates the surplus to an \( H_B \) type from announcing \( L_B \) and announcing \( H_B \) or

\[ 24 \text{ For “large enough” values of } D, \text{ the price of the } H_B \text{ bundle implied by this analysis would be so high that } H_B \text{ types would rather announce } L_B \text{ (i.e., violate (6)).} \]
Finally check to see that with
\[ H_a = aa \]
required fractional ticket allocation to those reporting
\[ tional cost of (i) constructing a random quantity-rationing mechanism to achieve the\]
delivering.

Risk-neutral customers. Assuming risk aversion would increase the profitability of intertemporal mixed bun-

ning. With intertemporal mixed bundling the seller need only be able to accept payment
requests and disburse tickets. Once tickets have sold out, the seller can stop processing
vendees. This implies that optimality
requires selling all units. Q.E.D.

The proof above, though somewhat laborious, highlights the relationship between
the intertemporal mixed-bundling allocation and that of the optimal mechanism design.
Note that \( L_{aa}^* = L_{bb}^* \) only when \( n_{LA} = n_{LB} \). This implies that \( D^* = 0 \) when \( n_{LA} = n_{LB} \)
or that the intertemporal mixed-bundling allocation is also the optimal allocation when
the distribution of customer types is symmetric.25 If \( L_{aa}^* > L_{bb}^* \) (i.e., when \( n_{LA} < n_{LB} \)),
then the \( H_b \) types have a greater willingness to announce their true type (buy the
bundle), as opposed to announcing they are low types, than do the \( H_A \) types, but the
single instrument of the period-1 bundle does not allow the seller to extract any more surplus from \( H_b \) types than from \( H_A \) types. Thus restricted to offering only bundles of
integer amounts, intertemporal mixed bundling maximizes the profit the seller can earn.

This analysis leads us to ask why we observe intertemporal mixed bundling but
not the more complex mechanism outlined above.26 The answers most likely lie in the
institutional details of implementing such a scheme. First, implementing the optimal
report mechanism will be more expensive than implementing intertemporal mixed bun-
dling. With intertemporal mixed bundling the seller need only be able to accept payment
requests and disburse tickets. Once tickets have sold out, the seller can stop processing
requests.27 To implement the direct-report mechanism, the seller must incur the additional
cost of (i) constructing a random quantity-rationing mechanism to achieve the
required fractional ticket allocation to those reporting \( H_A \) and (ii) identifying those reporting \( H_A \) that do not get a unit of \( B \) and refunding them \( v_2 \).28 It is then possible
that the additional cost of constructing and implementing the randomizing mechanism
would exceed the incremental revenue a seller could earn by implementing the optimal

\[ (v_4 - v_2)(L_{ab}^* + D^*)/n_{LA} = v_4 + v_2 - [v_4(1 + D^*/n_{HA} - L_{aa}^*) + v_3L_{ab}^* + v_2(1 - D^*/n_{HA})], \]

which simplifies to
\[ D^* = [(v_4 - v_2)(L_{aa}^* - L_{bb}^*)]/(v_4 - v_3)n_{LA} + (v_4 - v_2)n_{HA}. \]

All of the above assumes that the capacity constraint is binding. We must fi-

nally check to see that with \( H_{aa} = H_{bb} = H_{ba} = 1, H_{ab} = 1 - D^*, L_{aa} = L_{aa}^*, \) and
\( L_{bb} = L_{bb}^* + D^* \), the first-order conditions with respect to \( L_{aa} \) and \( L_{bb} \) are nonnegative,

\[ \frac{\partial \Pi}{\partial L_{aa}} = -n_{HA}(v_4 - v_3) + n_{LA}v_3 \]
\[ \frac{\partial \Pi}{\partial L_{bb}} = -n_{HB}(v_4 - v_3) + n_{LB}v_3, \]

which can be shown to be positive using (1) and (2). This implies that optimality
requires selling all units. Q.E.D.

The proof above, though somewhat laborious, highlights the relationship between
the intertemporal mixed-bundling allocation and that of the optimal mechanism design.

\[ 25 \text{ When the distribution is symmetric, all four incentive-compatibility constraints on } H \text{ types are just binding for } H_{ij} = 1 \text{ for all } i \text{ and } j. \text{ That is, an } H \text{ type is indifferent between announcing } H_i \text{ and } L_i \text{ as well as indifferent between announcing } H_i \text{ and } H_j. \]

\[ 26 \text{ We thank an anonymous referee for pointing out that applying our notion of frenzy suggests that empirically we should observe sellers offering bundles of the form } \text{“receive ticket } B \text{ with probability } 1 \text{ but receive ticket } A \text{ with a given probability } \lambda < 1.” \]

\[ 27 \text{ When tickets are sold individually, the seller takes orders by phone and sells tickets at specific locations at which customers queue up. Since people have an option, those who choose to queue up presumably have a low opportunity cost of their time, so the cost to society is likely to be small.} \]

\[ 28 \text{ The seller could charge all } H_i \text{ types a bundle price equal to the ex ante expected value of announcing } H_i \text{ and give no refunds. If customers are risk averse, they would be unwilling to pay this price. We assumed risk-neutral customers. Assuming risk aversion would increase the profitability of intertemporal mixed bundling.} \]

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mechanism as opposed to using intertemporal mixed bundling. This would be particularly true if the distribution of customer valuations were nearly symmetric, where the difference between total revenue from the optimal mechanism and the revenue from mixed bundling is small.

Perhaps a more important reason is that implementing the optimal direct-report mechanism imposes a cost of educating customers about how the system works. Management companies we spoke with indicated that it took three to four years of running subscription series before customers understood the mechanics of the selling mechanism (i.e., customers would try to order only some of the tickets in a series or order tickets not in the series, etc.). If a simple contract in which a customer submits a specified payment and receives a specified set of tickets takes three or four years for the public to understand, then a contract that delivers one ticket for sure and one ticket with some probability is likely to require substantial additional time for the public to understand.

Thus we can view the equilibrium as an optimal strategy when the set of contracts that can be offered is restricted by transactions costs. In particular, institutional cost factors and commitment problems restrict the seller to quantity ration only when his capacity constraint binds. While Proposition 1 shows that intertemporal mixed bundling dominates selling units individually, it is a simple exercise to show that it dominates any other selling strategy employing a combination of bundles and individual goods (for example, selling only bundles in period 1 and only bundles in period 2).

The intuition behind why the optimal design requires bundling (and why in the restricted game intertemporal mixed bundling is optimal) can be described as follows. If the seller could distinguish customer types, profits would be maximized by offering each $H_i$ type one unit of good $i$ at a price of $v_{i4}$ on a take-it-or-leave-it basis and offering each $L_i$ type one unit of good $i$ at a price of $v_{i3}$ on a take-it-or-leave-it basis. But because he cannot distinguish among types, it is impossible to make both of these offers concurrently. If he did, the $H_i$ types would accept offers made to the $L_i$ types.

One way to sell to the $H_i$ types at $v_{i4}$ would be to make no offers to the $L_i$ types, thus selling only at the highest price and throwing away all the remaining units. But (1) says this would not be as profitable as selling the entire capacity at $v_{i3}$. This means units sold to the $L_i$ types have positive marginal revenue. Thus, no strategy of selling less than one unit to each $L_i$ type (in expectation) and throwing the remaining units away can be profitable. Whenever marginal revenue is monotonic (starting with $H_i$’s and $L_i$’s each being offered one unit), every unit with positive marginal revenue that is removed from the $L_i$ allocation and thrown away reduces the revenue from the $L_i$ types by more than it increases the revenue that can be obtained from the $H_i$ types. In general, a unit with positive marginal revenue has positive marginal revenue in expectation. (See, for example, Pratt and Zeckhauser (1989) and Bulow and Roberts (1989).)

A strategy of reducing the allocation to each $L_i$ type could be profitable if instead of throwing away the unsold units, the seller could sell these units (1) to someone who valued them “sufficiently highly” and (2) in a way that would not cause an $H_i$ type to reduce his willingness to pay for the unit of good $i$ he is offered. One way to do this is to offer some of $i$ to each $H_{-i}$ in a bundle along with his unit of $i$. This will be profitable if the $H_{-i}$ type values a unit of $i$ highly enough, which occurs when $v_{i2}$ is close to $v_{i3}$. Of course only one unit of $i$ can be sold to each $H_{-i}$ profitably. Additional units are valued by the $H_{-i}$ at zero, so allocating more than one unit would be equivalent to throwing those units away.

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29 See the Appendix for a discussion of the cost of setting up a randomization mechanism.
A second possibility would be to sell some $i$ to $L_{-i}$ types. But this suffers from two problems. First, if the $L_{-i}$'s valuation is low, such an allocation would be “almost equivalent” to throwing a unit away. Second, and more generally, if prices are such that an $H_i$ is indifferent between announcing $H_i$ and $L_i$, adding units of $-i$ to the allocation of an $L_i$ would reduce an $H_i$'s willingness to pay for the $H_i$ allocation.

This explains why intertemporal mixed bundling is more profitable than selling units individually, and in the case of a symmetric distribution, why it is also the optimal selling strategy. With an asymmetric distribution there is an additional step. Start with the intertemporal mixed-bundling allocation and prices. When each $H_i$ type receives one unit of each good, each $H_i$ type is indifferent between announcing $H_i$ and $L_i$. In addition, prices are such that an $H_B$ strictly prefers announcing $H_B$ to $L_B$. The seller would like to raise the price to an $H_B$ but he cannot, because each $H_B$ would announce $H_A$. To increase this price, the seller must remove some of $B$ from the $H_A$ types’ bundle and add it to the $L_B$ allocation.\footnote{As before, an allocation to $H_B$ types would be like throwing units away, and allocating them to $L_A$ would likewise be unprofitable.} Since the compatibility constraint (5) is not binding on $H_B$ types, the addition to the $L_B$ allocation does not reduce the $H_B$ types’ willingness to pay, but it does increase the revenues from the $L_B$ types. Removing some $B$ from the $H_A$ allocation increases the revenue that can be extracted from the $H_B$'s while reducing the revenues from the $H_A$'s. The sum of the increased revenues from $L_B$'s and $H_B$'s exceeds the loss of revenue from the $H_A$'s. Once the price to the $H_B$’s has been raised enough to make an $H_B$ indifferent between announcing $H_B$ and $L_B$, no such further profitable reallocation is possible.

6. Extensions

We now provide two extensions of the model. First, we show that Proposition 1’s results can be extended to a market with continuous distributions of valuations, demonstrating that frenzy equilibria are not an artifact of discrete distributions. Second, we extend our results to a situation where the seller offers more than one bundle and each bundle contains a high-demand good and a low-demand good, rather than all of the highest-demand goods being bundled together.

With a continuous demand curve for a single good and nonrandom demand, a seller in a two-period market would never set a price in period 2 such that there would be excess demand.\footnote{This assumes there is no incentive to develop a reputation for creating excess demand, as there might be in an infinitely repeated game.} That is, once the seller observed the residual demand curve in period 2, he would have the incentive to set a price so that the market cleared. Thus, in order to generate a buying frenzy based on the credible threat of period-2 rationing, the seller must be able in period 1 to commit to a period-2 price.\footnote{With no commitment to a period-2 price, high-type customers might buy a bundle to avoid high market-clearing period-2 prices. The bundle price can’t exceed the sum of the period-2 prices. The seller could benefit if the period-2 prices exceed the market-clearing prices that would prevail in the absence of bundling.}

In the Appendix we present a model in which the capacity-constrained seller faces demand curves for each of two goods, $A$ and $B$, that are linear. The $H_i$ types occupy the upper segment of the demand curve for good $i$ and the segment of the demand curve for $-i$ just to the right of the seller’s capacity constraint. $L_i$ types occupy the segment of the demand curve between the $H_i$ segment and the capacity constraint on the good $i$ demand curve, and the bottom of the $-i$ demand curve. The location of
each customer on his type’s segment on the $i$ demand curve is perfectly negatively correlated with his location on his type’s segment on the $-i$ demand curve.

Given this structure, the equilibrium bundle price is derived in the same way as the bundle price in Proposition 1. In particular it equals the value of the bundle to an $H$-type customer less the maximum expected surplus an $H$ type could earn if he waited until period 2 to purchase, when all other $H$ types purchase a bundle.

The fact that intertemporal mixed bundling is profitable when the demand curves for the goods are linear suggests that there need not be any unusual characteristics in the overall market demand curves for this strategy to be profitable. What is needed are characteristics pointed out in the examples with discrete valuation, mainly that the highest-valuation customers for one good have a valuation for the other good that is close to but below that good’s individual good market-clearing price. Again the intuition behind this condition is that the cost to the seller of inducing rationing is the opportunity cost of selling some units in the first period at a price below the market-clearing level. The closer the value of these units to the market-clearing level, the lower the opportunity cost of using intertemporal mixed bundling, and therefore the more likely this is to be profitable.

There are three interesting observations about the use of subscription series in the market for summer concert tickets. The first is that there are often low-demand acts bundled with high-demand acts.33 Popular lore suggests that this is a way to sell tickets for low-demand acts, since people who “will pay anything” to see a high-demand act will buy the low-demand act for a price above its valuation. This reasoning is not completely convincing, since if people will “pay anything” for the high-demand act, then the seller can “charge anything” for the act without bundling it.34

The second observation is that virtually all amphitheaters that use intertemporal mixed bundling have two or three subscription series consisting of different acts during a single season. The third is that while in most amphitheaters the bundle price exceeds the sum of the individual prices, there is one amphitheater for which this is not true.

If our theory provides an explanation for the use of bundling in this market, it must be able to generate these results. We outline an example that can generate these three observations. There are four goods, $A$, $B$, $C$, and $D$, and eight types of customers, a high- and a low-value type for each of the four goods. A high-value customer for good $i$ will pay $250 for a unit of good $i$ and $100$ for a unit of any other good. A low-value customer for good $i$ will pay $150$ for one unit of that good and $50$ for a unit of any other good. Capacity is 100 for each good. There are 50 $H_A$’s, 50 $H_B$’s, 25 $H_C$’s, 25 $H_D$’s and 50 of each low-type customer, so that $A$ and $B$ are considered high-demand goods.

In a frenzy equilibrium the seller sells 75 bundles consisting of (without loss of generality) one unit of $A$ and one unit of $C$, and 75 bundles consisting of one unit of $B$ and one unit of $D$ at a price of $300$, and then sells the remaining 25 units of each good at a price of $150$. His payoff is $60,000$. This exceeds the $57,500$ that could be earned by selling 100 bundles of $A$ and $B$ at $350$ and selling 75 units each of $C$ and $D$ at $150$. This establishes our first observation. The $60,000$ payoff also exceeds the $55,000$ the seller could earn in the frenzy generated by offering only bundles.

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33 That is, there are acts not included in the bundle that are more popular than acts that are included in the bundle.

34 Thaler (1985) argues that such bundling makes the prices charged seem more fair than charging a price for a popular act that is significantly higher than the “reference price” of other acts. However, we note that the Rolling Stones have played concerts where the individual ticket prices ranged between $300$ and $400$. 

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containing one unit each of \( A, B, C, \) and \( D \) for a price of $550. This establishes our second observation.

The third can be established by noting that the period-1 bundle price is equal to the sum of the period-2 individual prices. The equilibrium price of a bundle is an increasing function of an \( H_i \) type’s valuation for good \( i \). The period-2 price is independent of an \( H_i \) type’s valuation of good \( i \). Thus, by assuming an \( H_i \) type’s valuation for good \( i \) exceeds $250, we can obtain a frenzy in which the bundle price exceeds the sum of the individual prices. Similarly, by assuming a valuation less than $250, we can obtain a frenzy in which the bundle price is less than the sum of the individual prices.\(^{35}\)

This example introduces a new complication by considering goods for which the individual profit-maximizing price causes the seller to sell fewer units than his capacity for low-demand goods. This generates a second incentive for using bundling, which is that the seller can sell more tickets using bundling than he can by selling goods individually.

To understand why the seller bundles high- and low-demand goods instead of both high-demand goods, it is useful to compare the incremental profits resulting from each strategy to those that would be earned by selling all units individually. If the seller bundles both high-demand goods, the resulting frenzy price allows him to capture all of the surplus of the \( H_i \) types for good \( i \), which amounts to $100 over the individual good market-clearing price for each of these units. However, selling a unit of \( i \) to an \( H_{-i} \) type involves a $50 opportunity cost relative to the market-clearing price. Thus each of the 100 bundles sold represents an increase in revenue of $50 per bundle, or a total of $5,000 above the market-clearing revenue. Since it is impossible to create a frenzy by bundling \( C \) and \( D \), there can be no additional revenue raised from bundling these goods.

A bundle consisting of a high-demand and a low-demand good sells for $300, which is equal to the sum of the individual market-clearing prices for the component units. Thus the frenzy extracts no additional surplus from the \( H \) types. The extra surplus extracted from an \( H_i \) type for buying good \( i \) is exactly offset by the opportunity cost incurred by selling an \( H_i \) type a unit of \( -i \). However, an additional 50 units are sold at the market-clearing price of $150, which increases the seller’s revenue relative to the individual market-clearing revenue by $7,500. This suggests that when the revenues from additional units to be sold exceed the additional surplus that can be extracted from a frenzy, the seller will choose to bundle high- and low-demand goods.

## 7. Conclusion

In this article we introduce mixed bundling in an intertemporal setting. We have shown that, unlike the static mixed-bundling analysis, intertemporal mixed bundling presents the possibility that the bundle price can be greater than the individual component prices. Such bundling extracts surplus from higher-value customers, and the individual units are sold to lower-value customers (in contrast to a static model of mixed bundling in which the bundles extract surplus from customers with low valuation for individual goods).

Intertemporal mixed bundling is interesting because it uses the threat of shortages in one market to credibly create a shortage in a second market. Thus shortages are caused by an externality that high-value customers impose on each other. Because each

\(^{35}\) Formal derivation of these results are available from the authors upon request.

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believes all other high-valuation types will purchase a bundle, it is rational for each to do so.

The seller wishes to induce a frenzy because it acts as a self-selection mechanism. The rationing that will occur when units are sold separately makes purchasing an individual unit an unattractive option relative to purchasing a bundle with certainty for high-value customers. This allows the seller to extract additional rents from the high-value customer by offering the bundle at a high price, one that exceeds the sum of the individual prices.

Appendix

Optimal direct-report mechanism, one good. The seller faces two types of buyers, high and low, which he cannot distinguish. He therefore offers two types of contracts. If a buyer announces he is a high type, the contract allocates to him \( x_H \) units and he pays \( P_H \) for this allocation. If the buyer announces he is a low type, the contract allocates \( x_L \) (in expectation) and he pays a price in expectation of \( P_L \). The seller is constrained by his capacity to satisfy \( (\frac{\gamma}{\gamma})x_H + (\frac{\gamma}{3})x_L \leq 1 \).

Low-value customers require a surplus of zero, so \( P_L = x_L \). High-value customers who report low would then receive \( 2x_L - x_H = x_L \) in surplus. To report truthfully, a high type must receive at least this surplus. Thus \( P_H = 2 + (x_H - 1)(1 - \epsilon) - x_L \).

The seller’s payoff is

\[
\Pi = (\frac{\gamma}{3})(2 + (x_H - 1)(1 - \epsilon) - x_L) + (\frac{\gamma}{3})x_L = (\frac{\gamma}{3})(2 + (x_H - 1)(1 - \epsilon) + x_L),
\]

and noting that \( (\frac{\gamma}{3})x_H + (\frac{\gamma}{3})x_L = 1 \), we can write \( x_L \) in terms of \( x_H \) as \( x_L = \frac{\gamma}{3} - (\frac{\gamma}{3})x_H \) to get

\[
\Pi = \frac{\gamma}{3} + (\frac{\gamma}{3})\epsilon + (\frac{\gamma}{3})x_H - (\frac{\gamma}{3})\epsilon x_H.
\]

When \( \epsilon < \frac{1}{2} \), the derivative of revenue with respect to \( x_H \) is \( (\frac{\gamma}{3}) - (\frac{\gamma}{3})\epsilon \) for \( 1 \leq x_H \leq 2 \), which is positive, which implies that revenue is maximized for \( x_H \geq 2 \). When \( x_H \geq 2 \), increasing the allocation \( x_H \) provides no additional benefit to \( H \) types, so \( \Pi \) can be rewritten as \( \Pi = (\frac{\gamma}{3})(2 + (1 - \epsilon) - x_L) + (\frac{\gamma}{3})x_H \).

The derivative with respect to \( x_H \) is negative, which implies \( x_H \leq 2 \). Therefore \( x_H = 2 \). When \( x_H = 2 \), the capacity constraint then implies that \( x_L = \frac{\gamma}{3} \) a unit in expectation. This is the allocation achieved by bundling.

The intuition can be cast as comparing the marginal revenue generated by marginally increasing the allocation \( x_H \) to high-type customers to the marginal revenue generated by marginally increasing the allocation \( x_L \) to the low-type customers. The marginal revenue for selling an extra unit to an \( H \) type (holding fixed the level sold to \( L \) types) for \( 1 < x_H < 2 \) is just the \( H \) type’s value for the second unit \((1 - \epsilon)\). (For \( x_H \geq 2 \), the marginal revenue is zero, since no \( H \) type values more than two units.)

The marginal revenue from increasing an additional unit to the \( L \) allocation (holding the \( H \)-type allocation fixed) is simply the sum of the value to low-value customers, which for allocations less than one is equal to one, and the reduction in the willingness to pay of high types if they announce their true type, because increasing the \( L \) allocation increases the incentive for a high type to report he is low type. In other words, the marginal revenue of an additional unit sold to a low type is the marginal valuation of the low type less the reduction in revenue suffered because the incentive-compatibility constraint is altered for the high types. This is multiplied by the mass of low types times the expectation of not being rationed.

The analysis above shows that for \( \epsilon < \frac{1}{2} \) and \( x_H < 2 \), the marginal revenue in the market for \( H \) types exceeds the marginal revenue from \( L \) types. Thus the seller should increase the allocation of \( x_H \) to two. Once \( x_H = 2 \), the marginal revenue of an additional allocation is zero. When \( x_H + x_L = 1 \) is binding, increasing \( x_H \) decreases \( x_L \), reducing the marginal revenue from units sold to low types by \( 2x_L \) without increasing revenue from units sold to high types. Thus the seller would never increase \( x_H \) above two.

Proof of Proposition 2. The following constitutes the relevant portion of a perfect Nash equilibrium.

Seller:

\[
P_p = v_1 + v_2 - (v_4 - v_3)(K - (x_H + x_L))h_1A.
\]

\( P_{2H} = P_{2L} = v_3 \) if no non-\( H \)-type customers purchase a bundle in period 1 or if no non-\( A \)-type customers

\footnote{The benefit he receives from \( x_H \) is \( 2 + (x_H - 1)(1 - \epsilon) \) for \( x_H > 1 \), and so \( P_H \) satisfies

\[2 + (x_H - 1)(1 - \epsilon) - P_H = x_L.\]}

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purchase a unit of good $A$, no non-$B$-type customer purchases a unit of good $B$ in period 1 and no $L_i$-type customer purchases a unit of $i$ in period 1 unless all $H_i$ customers purchase a unit of $i$ in period 1.

$H$-type customers:

\[
\begin{align*}
R_{\text{HA}} &= R_{\text{HB}} = v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA} \\
R_{\text{AIA}} &= R_{\text{AIB}} = v_3, \quad R_{\text{AILA}} = R_{\text{BIBA}} = v_2 \\
R_{\text{AILA}} &= R_{\text{AIB}} = v_3, \quad R_{\text{AILA}} = R_{\text{BIBA}} = v_2.
\end{align*}
\]

$L$-type customers:

\[
\begin{align*}
R_{\text{LA}} &= R_{\text{LB}} = v_3 + v_i \\
R_{\text{AILA}} &= R_{\text{AIB}} = v_3, \quad R_{\text{AILA}} = v_i \\
R_{\text{AILA}} &= R_{\text{AIB}} = v_3, \quad R_{\text{AILA}} = v_i.
\end{align*}
\]

The payoffs are as follows.

Seller:

\[
[v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}][2(n_{HA} + n_{HB}) + 2(K - (n_{HA} + n_{HB}))v_i]
\]

$H_a$ and $H_b$:

\[
(v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}.
\]

$L_a$ and $L_b$:

\[
0.
\]

Lemma A1. Given that the seller will set period-2 prices equal to $v_i$, selling bundles in period 1 at a price of $v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}$ is a best response to all of the customers’ strategies.

Proof. Given customers’ period-2 reservation prices, the most that the seller could earn by selling goods individually is $2Kv_i$, which is less than

\[
[v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}][2(n_{HA} + n_{HB}) + [2(K - 2(n_{HA} + n_{HB})v_i]
\]

whenever the condition of the proposition holds. Thus, selling bundles in period 1 at a price of

\[
v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}
\]

earns a higher payoff for the seller than selling the goods individually in period 1. Setting the bundle price at $v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA}$ also yields a higher payoff to the seller than any other bundle price. At any bundle price higher than this, the seller sells no bundles in period 1 and sells all units in period 2, earning a payoff of $2Kv_i$. For bundle prices between

\[
v_4 + v_2 - (v_4 - v_3)(K - (n_{HA} + n_{HB}))n_{LA} \quad \text{and} \quad v_3 + v_i,
\]

the seller sells no additional bundles but earns a lower revenue per bundle. At a price at or below $v_i + v_j$, the seller sells all his capacity in period 1, earning a payoff less than or equal to $K(v_i + v_j)$.

We now show that each customer’s strategy constitutes a subgame-perfect best response to the seller and all other customers’ strategies. For the seller, setting the individual period-2 prices equal to $v_i$ is a best response to customers’ period-2 reservation prices if either (i) no non-$H$-type customers purchase in period 1 or (ii) no non-$A$-type customer purchases a unit of good $A$, no non-$B$-type customer purchases a unit of good $B$ in period 1, and no $L_i$-type customer purchases a unit of $i$ in period 1 unless all $H_i$ customers purchase a unit of $i$ in period 1.

Lemma A2. In period 2 it is a weakly dominant strategy for every customer to set his reservation price equal to his true valuation for any good he has not purchased in period 1.

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Proof. If a customer does not purchase a unit of a good, the increment to his payoff from that unit is zero. If he purchases at a price equal to his valuation, the increment is also zero. If he purchases at any price less than his valuation, the increment to his payoff is positive.

Lemma A3. Given that all customers in period 2 will purchase a unit of a good if it is priced at or below their valuation for the unit, the payoff-maximizing period-2 price for each good is \( v_i \) whenever no non-\( H \) types purchase a unit in period 1.

Proof. The assumption that \((n_{HA} + n_{HB})v_i < 2Kv_i\) implies that if no units are purchased in period 1, the seller earns a higher payoff by selling all units at a price of \( v_i \) than by selling \( n_{HA} + n_{HB} \) units at a price of \( v_i \). If \( H \) types purchase bundles in period 1 and no non-\( H \) types purchase, then the payoff from selling goods individually in period 2 at \( v_i \) is \( 2(K - z) \), which is greater than the payoff from selling goods at a price of \( v_i \), which would be \((n_{HA} + n_{HB} - z)v_i\).

Lemma A4. Given that all customers in period 2 will purchase a good if it is priced at or below their valuation for the good, the payoff-maximizing period-2 price for each good is \( v_i \) if no non-\( A \)-type customer purchases a unit of good \( A \). no non-\( B \)-type customer purchases a unit of good \( B \) in period 1, and no \( L \)-type customer purchases a unit of \( i \) in period 1 unless all \( H \) customers purchase a unit of \( i \) in period 1.

Proof. When these conditions are met, the seller can sell all of his remaining units in period 2 at a price of \( v_i \). If all \( H \) customers have purchased bundles, then there are no customers with reservation prices above \( v_i \), so this must be the payoff-maximizing price. If no non-\( i \)-type customers have purchased a unit of \( i \) in period 1, then the assumption that \((n_{HA} + n_{HB})v_i < 2Kv_i\) implies that \( v_i \) is the payoff-maximizing price.

Lemma A5. Given that both goods will be priced at \( v_i \) in period 2, each \( H \) type can do no better than setting \( R_{HA} = v_i \) and \( R_{HB} = v_i \). Similarly each \( L \) type can do no better than setting \( R_{LA} = v_i \) and \( R_{LB} = v_i \).

Proof. If goods are sold individually in period 1 for any price greater than \( v_i \), and each customer sets his reservation prices as listed above, there will be no excess demand in period 2, which means any customer with valuation for a good greater than or equal to \( v_i \) will be able to purchase that good with certainty in period 2 at a price of \( v_i \). Thus no customer could ever improve his payoff by setting a period-1 reservation price above \( v_i \). Similarly, since a customer with valuation above \( v_i \) can purchase with certainty in period 2 at a price of \( v_i \), he cannot improve his payoff by lowering his period 1 reservation price below \( v_i \). In fact, such a deviation would not be subgame perfect because setting a reservation price at any \( R'_{HB} \) below \( v_i \) in period 1 would not yield the maximum payoff in any subgame defined by \( P_{HB} \) such that \( v_i > R'_{HB} > R'_{LA} \).

Lemma A6. It is a best response for an \( H \)-type customer to set \( R_{HA} = v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA} \) when all other \( H \)-type customers have done the same, and the seller sets a period-1 bundle price of

\[
\frac{v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA}}{n_{LA}}.
\]

Proof. If an \( H \)-type customer chooses a reservation price for the bundle below

\[
v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA}
\]

when all other \( H \) types have set theirs equal to it, then in period 2 he will be able to purchase a unit of \( A \) for a price of \( v_i \) with probability \((K - (n_{HA} + n_{HB}))\mid n_{LA}\), yielding an expected payoff of

\[
(v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA}.
\]

This is the same payoff he would receive if he set his reservation price at

\[
v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA}.
\]

Thus he cannot improve his payoff by setting a reservation price for the bundle below

\[
v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA}.
\]

In every subgame defined by the seller charging a bundle price less than

\[
v_i + v_2 - (v_i - v_3)(K - (n_{HA} + n_{HB}))\mid n_{LA},
\]

an \( H \)-type customer’s best response is to purchase a bundle if all other \( H \) types purchase a bundle as well.

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Similarly, an $H$ type could not improve his payoff by setting a period-1 bundle reservation price above $v_i + v_j - (v_i - v_j)(K - (n_HA + n_HB))n_{LA}$, because he would still purchase the bundle in period 1. (Setting a reservation price at $K$ would not constitute a best response in subgames where all other $H$ types set their reservation prices equal to

$$v_i + v_j - (v_i - v_j)(K - (n_HA + n_HB))n_{LA}$$

and the seller charged a period-1 bundle price between $K$ and $v_i + v_j - (v_i - v_j)(K - (n_HA + n_HB))n_{LA}$.

Lemmas A2 through A6 imply that the strategies for $H$-type customers described constitute subgame-perfect best responses.

**Lemma A7.** $L$-type customers can do no better than to set their period-1 reservation prices for both individually sold goods and bundled goods equal to their valuation for the goods.

**Proof.** Given the strategy of the seller and the $H$-type customers, the $L$-type customers earn a zero payoff by setting reservation prices equal to their valuations. An individual $L$ type cannot improve his payoffs by lowering any of his reservation prices, since given the seller’s strategy, this would not result in purchasing a good (or a bundle) at a price below his valuation for that good (or bundle). (Setting a reservation price strictly below his valuation would not be a best response in a subgame defined by the seller setting a price between his true valuation and his reservation price.) Similarly, an $L$ type could only reduce his payoff by setting a reservation price above his valuation, since any purchase he made at a price above valuation would result in a negative surplus. *Q.E.D.*

**□**

**Proof that** $L_{n_i} = 0$. Consider the allocation of Proposition 2 in which $L_{n_i} = 0$ and ask if profits can be increased by allocating some units of $-i$ to $L_{n_i}$. There are three cases to consider.

(i) Suppose $D$ units are allocated from $H_{n_i}$ to $L_{n_i}$. This reduces revenue from the $H_{n_i}$ types by $Dv_i$ and increases revenue from $L_i$ types by $Dv_i$. This must always reduce revenue.

(ii) Suppose $D$ units are allocated from $L_i$ to $L_{n_i}$. This reduces revenue from the $L_i$ customers by $Dv_i$ and increases revenue from $L_{n_i}$ types by $Dv_i$. For $i = A$, removing $D$ units from $L_{LA}$ and allocating them to $L_{n_i}$ allows the seller to increase the price to $H_{n_i}$ types but requires that it lower the price to $H_A$ types. The allocation decreases the value of announcing $L_A$ to an $H_A$ by $(v_i - v_j)n_{LA}$. This allows the seller to increase the price of the bundle to the $H_{n_i}$ types by this amount, increasing the seller’s revenue by $n_{HA}(v_i - v_j)Dn_{HA}$. The allocation also increases the value of announcing $L_A$ to the $H_A$ types, so the seller must reduce the price of the $B$ bundle by $(v_i - v_j)Dn_{LB}$, decreasing his revenue by $n_{HB}(v_i - v_j)Dn_{LB}$. Thus the change in revenue to the seller is $n_{HA}(v_i - v_j)Dn_{HA} + Dv_i - n_{HB}(v_i - v_j)Dn_{LB}$. Condition (1a) along with (2) ensure that the sum of the first two terms is negative, and (3a) and (2) ensure that the sum of the third and fourth terms is negative. Thus this reduces the seller’s profit.

(iii) Suppose $D$ units are allocated from $H_{n_i}$ to $L_{n_i}$. This increases revenue from $L_i$ by $Dv_i$, it increases the value to announcing $L_i$ to $H_{n_i}$ customers by $(v_i - v_j)Dn_{LA}$, so the seller’s revenue would fall (at least) by $n_{HB}(v_i - v_j)Dn_{LB}$, resulting in a change in the seller’s revenue of $Dv_i + n_{HB}(v_i - v_j)Dn_{LB}$, which (3) implies is negative. *Q.E.D.*

**□**

**First-order conditions for Proposition 2.** Ignoring for the moment (6), the first-order conditions for the allocation levels are

$$\frac{\partial \Pi}{\partial H_{n_i}} = n_{HA}v_i - n_{HA}(v_i - v_j)\frac{\partial L_{n_i}/\partial H_{n_i}}{\partial H_{n_i}} + n_{LA}v_i\frac{\partial L_{n_i}/\partial H_{n_i}}{\partial H_{n_i}} \quad \text{for } H_{n_i} \leq 1$$

$$- n_{HA}(v_i - v_j)\frac{\partial L_{n_i}/\partial H_{n_i}}{\partial H_{n_i}} + n_{LA}v_i\frac{\partial L_{n_i}/\partial H_{n_i}}{\partial H_{n_i}} \quad \text{for } H_{n_i} > 1$$

$$\frac{\partial \Pi}{\partial H_{n_0}} = n_{HA}v_i - n_{HA}(v_i - v_j)\frac{\partial L_{n_0}/\partial H_{n_0}}{\partial H_{n_0}} + n_{LA}v_i\frac{\partial L_{n_0}/\partial H_{n_0}}{\partial H_{n_0}} \quad \text{for } H_{n_0} \leq 1$$

$$- n_{n_0}(v_i - v_j)\frac{\partial L_{n_0}/\partial H_{n_0}}{\partial H_{n_0}} + n_{LA}v_i\frac{\partial L_{n_0}/\partial H_{n_0}}{\partial H_{n_0}} \quad \text{for } H_{n_0} > 1$$

where $\frac{\partial L_{n_i}/\partial H_{n_i}}{\partial H_{n_i}} = -n_{HA}/n_{LA}$ and $\frac{\partial L_{n_0}/\partial H_{n_0}}{\partial H_{n_0}} = -n_{HA}/n_{LA}$ if the capacity constraint on $L_{n_i}$ is binding and zero if the constraints are not binding, and

---

37 If the constraint is that $H_{n_i}$ is indifferent between announcing $H_i$ and $H_{n_i}$, then this would also require the seller to decrease the price of the bundle to $H_{n_i}$ customers as well.

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such recontracting would involve significant incremental transactions costs beyond those that would be in-
market demand curve of the form 

$\text{refund} v
$

possible, then inducing a frenzy would be impossible. If

$B$
sell a unit of

$A$
since they could wait until period 2 and purchase the unit for

$v
$

therefore,

$\text{Lin} H_{bb} \text{ LB} = n_{lin} + 1$ assumed that

$H_{ba}$

value it at

$v
$

2 to the fraction of those claiming to be

$v
$

which is the individual price profit-maximizing quantity. 38

Discussion of costs of implementing the optimal direct-report mechanism. A large part of the cost of randomizing is making the process public and verifiable to ensure that $H_a$ types receive a unit of $B$ with the probability implied by the direct-report mechanism. If it is not verifiable, it may be difficult for the seller to implement such a scheme credibly. Once an $H_a$ type has identified himself, the seller has no incentive to sell him a unit of $B$, since the $H_a$ types only value the good at $v_1$ when there are unsecured $L_a$ types who value it at $v_1$. Given that the seller has incurred the sunk costs to set up the administrative apparatus to refund $v_1$ to the fraction of those claiming to be $H_a$ indicated by the optimal mechanism, he has the incentive to refund $v_1$ to all $H_a$ types and sell all of the retained units of $B$ at a price of $v_1$. If such “recontracting” were possible, then inducing a frenzy would be impossible. If $H$ types knew it was not credible for the seller to sell a unit of $B$ to any of those announcing $H_a$, then no $H_a$ would have an incentive to purchase a bundle, since they could wait until period 2 and purchase the unit for $v_1$ with certainty. With no $H_a$ types purchasing

$A$

there is no reason for $H_a$ types to purchase a bundle in period 1, so the frenzy cannot be sustained.

The intertemporal mixed-bundling mechanism could also be subject to potential recontracting. However, such recontracting would involve significant incremental transactions costs beyond those that would be incurred to set up the apparatus to implement bundling. One reason is that under intertemporal mixed bundling, $H_a$ types are not distinguished from $H_b$ because their purchasing behaviors are identical. So to engage in recontracting, the seller would have to actively distinguish the $H_a$ types from the $H_b$ types, an activity that would require the seller to incur additional costs.

Linear example (from Section 6). We again consider two goods, $A$ and $B$. Each good has a linear market demand curve of the form $P_i = 100 - Q_i$ for $i \in \{A, B\}$. The seller is capacity constrained at $K = 50$, which is the individual price profit-maximizing quantity. 38

$\text{Q.E.D.}$

38 The analysis will hold for $K < 50$. © RAND 1999.
Each customer values one unit of each good. Let \( j_i \) be the location of customer \( j \) along the horizontal axis in market \( i \). An \( H_A \) type is a customer located at \( j_A \) between zero and \( J < 25 \) on the horizontal axis in market \( A \) and therefore has a valuation for good \( A \) equal to \( 100 - j_A \). Similarly, an \( H_B \) type is located at \( j_B \) between zero and \( J \) on the horizontal axis of market \( B \) and values a unit of \( B \) at \( 100 - j_B \). An \( H_i \) type is located at point \( 50 - J + j_i \) on the horizontal axis for good \(-i\) of \( 50 - J + j_i \). Thus the \( H_i \)-types occupy the “top” of the demand curve in market \( A \) and the segment of the demand curve in market \( B \) that is just to the right of the seller’s capacity constraint. The \( H_i \)-types similarly occupy the top of the demand curve in market \( B \) and the segment of the demand curve for \( A \) that is just to the right of the capacity constraint. This structure also implies that the valuations are perfectly negatively correlated, so every \( H \) type values a bundle at \( 150 - J \).

The \( L_A \) and \( L_B \) types fill in the rest of the demand curves in an analogous manner. An \( L_A \) type has location \( j_A \) for \( J < j_A < 50 \) on the good \( A \) horizontal axis and therefore has a valuation of \( 100 - j_A \) for good \( A \). An \( L_A \) type has a location \( 100 - J + j_A \) on the good \( B \) horizontal axis and therefore has a valuation of \(-J + j_A \) for good \( B \). \( L_a \) types are distributed in a similar manner. Table A1 summarizes the valuations for each of the four types. Figure A1 depicts the locations of the four types of customers in market \( A \).

**Observation 1.** The seller can earn a higher profit by using intertemporal mixed bundling than he can by selling goods individually.

**Proof.** If the goods are sold individually, then the profit-maximizing strategy for the seller is to set a price for each good of \( 50 \). It is well known that with linear demand curves, the seller cannot earn a higher profit by selling goods with certainty at a period-1 price above \( 50 \) while selling goods with quantity rationing at
a period-2 price below $50. So if he sells the goods separately, the seller earns a maximum profit in the two markets of $5,000.

But if the seller committed to a period-2 price of $50 and offered a bundle in period 1, then a buying frenzy equilibrium exists in which all of the \( H \) types buy a bundle at a price of 100 + \( J/(50 - J) \) in the first period. An \( H \) type values the bundle at 150 - \( J \) and the \( H \) type that values good \( i \) the most (i.e., at 100) would receive an expected surplus of 50/[1 - (\( J/(50 - J) \)] if he waited until period 2 while all other \( H \) types purchased a bundle. (For \( J < 25 \) the bundled price exceeds 100). In such a frenzy the seller would earn a payoff of 5000 + 2\( J/(50 - J) \), which for \( J \leq 25 \) is greater than 5,000. \( Q.E.D. \)

The results of this example can be interpreted as a direct extension of those in Proposition 1. \( ^{41} \) \( n_{HA} \) and \( n_{JA} \) and \( n_{LH} = 50 - J \). The individual market-clearing price is 50, which is the price at which the demand of all \( H \) and \( L \) customers equals the capacity of good \( i \). As in Proposition 1, this is also the period-2 price in the frenzy equilibrium.

**References**


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41 The only real difference between the two models is that in Proposition 1 each member of a customer type has the same valuation for each component good. In this example, each member of a customer type has the same valuation for the bundle but a valuation for each good in a specified range.

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