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Interdependencies among brands in a firm's product line should be considered when marketing strategies are formulated. This article develops a mathematical model of the interaction among products for normative strategy recommendations. An empirical example applying the a priori model to a product line decision suggests that the model prespecification is relevant and useful.

A Mathematical Modeling Approach to Product Line Decisions

Most firms market several somewhat similar products called a product line. Policies of product diversification and new product introduction have been implemented by widening the product line. Depth in the product line has emerged as firms attempt to meet competition and satisfy the needs of the market's subsegments. Although the multiproduct firm has grown in importance, there has not been a corresponding growth in model building and research to help solve the marketing problems of firms with a product line.

Product line decisions are difficult because the products in the line are not usually independent. Products cannot be optimized individually and then added to the line to produce optimum product line results. The marketing mix established for one product may affect the sales of another product; interdependency is the key consideration in product line decision making. This article develops a mathematical product line model that analyzes the marketing strategy implications of product interdependency. The model will be developed by a priori reasoning and will be subjected to a preliminary test based on empirical market data.

A MATHEMATICAL MODEL OF THE PRODUCT LINE DECISION

Model Development Criterion

In developing a model of product line effects many approaches are available ranging from micro-analytic simulation and its potential for a highly disaggregated consideration of the consumer choice process[1] to a highly aggregated model that might be represented in a simple linear regression model. Between these extremes there are several other levels of aggregation such as complex single equation and multiple equation models.

In developing the model proposed in this article, two criteria were established. The first was decision relevance, i.e., the model should encompass the major factors and market phenomena affecting the problem of finding the best marketing mix for a product line. The second criterion is reflected in the doctrine of parsimony and requires that simple models be preferred whenever possible. These criteria suggest the development goal that the product line model should be the simplest model that encompasses the relevant market phenomena and is useful in decision making.

To specify the relevant phenomena the basic consumer choice process should be examined.[2] For example, consider the purchasing process for four classes of goods related to shaving: electric shavers, safety razors, aerosal shave cream, and after shave lotion. The consumer choice process originates with the development of an awareness of these classes of goods and the particular brands in these classes. Awareness may be produced by advertising, personal selling, word of mouth, or post-buying experience.

The consumer also forms attitudes about each product class, the relationship between these classes, and the brands in the classes. These attitudes may be directed toward product characteristics or advertising appeals. For example, consumers will form attitudes

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[1] See [1].

[2] This brief description is consistent with Nicosia's basic structure. See [9].

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about electric shavers and their advertising appeals. They will also form attitudes about electric versus saftey razors, and electric razors versus after shave lotion. These consumer attitudes become prime factors for developing a perceived need for the products. When the level of perceived need is sufficient, a search effort —a shopping trip in this example—is conducted and a purchase decision is made. In the store the consumer is influenced by point of purchase communication, and he integrates this new information with existing attitudes. The consumer will choose the product with the greatest perceived utility per dollar, assuming a satisfactory alternative is present. It can be expected that the consumer's willingness to buy a product at a given price will depend on his attitude toward the product's characteristics and appeals. This implies a marketing mix effect between price and advertising since price response will depend on the level of advertising.

In examining the purchase decision the effects between and within product classes should be considered. Since the proneness to purchase from a group is a function of the perceived need for that group, the combined advertising of all brands in the group may influence the attitudes and utility ascribed to the product class. For example, if all brands of after shave lotion increase their advertising, the attitudes and sales of the group may increase. The same product group phenomena could occur for other product classes, such as safety razors and electric razors. However, in these cases the additional sales generated by a group may be obtained from a related product group. For example, if all safety razor brands increase their advertising, consumers may develop attitudes that suggest substitution of safety razors for electric shavers. Substitution is not the only possible intergroup effect; some groups may be complementary. If advertising of safety razors increases, the perceived need for shaving cream and perhaps after shave lotion may increase.

An intragroup phenomenon, the competition of brands within the group, also exists. If consumers choose a brand because of relative utility per dollar, the selection reflects a combination of attitudes and price and implies that the relative marketing mix effects of brands are significant. For example, the in-store choice of Schick over Gillette would probably reflect the combined advertising, promotion and price advantage of Schick over Gillette as perceived in the customer's utility assessment.

From this brief and simplified consideration of the process, an a priori specification of the most important phenomena can be derived. For the model three factors were identified as having high behavioral and decision relevance: (1) aggregate product class marketing mix effects, (2) product class interdependencies, and (3) intragroup relative competitive brand effects. The goal was to design a simple model of these phenomena for aiding product line decisions.

Aggregate Product Class Marketing Mix Effects

For a simple model of the combined effects of price and nonprice marketing activity, an aggregate sales response function will be postulated. This function should include three basic marketing variables: advertising, price, and distribution. Distribution may be measured by the percent of outlets carrying the product, the number of salesmen selling the product, the middleman's margin on the product, or a combination of these. The simplest form would be a linear equation of these variables, but this has two disadvantages. First, it does not allow marketing mix effects since the sales response to a variable is not affected by other variables. Second, the linear form would imply a linear response to advertising which can lead to unreasonable decision implications since it usually implies extremes of large or small levels of advertising. A linear form would also not allow for decreasing effects on advertising expenditure.[3] The next most appropriate form for representing the mix effects is a linear log function. In unlogged form the formulation would be:[4]

$$X_{jI} = a\, P_{jI}^{EPI}\, A_{jI}^{EAI}\, D_{jI}^{EDI} \tag{1}$$

X_{jI} is industry sales of Product j
a is scale constant
P_{jI} is average price level of all brands in product group j
A_{jI} is total advertising of all brands in product group j
D_{jI} is total distribution level for all brands in product group j
EPI is industry price elasticity for Product j
EAI is industry advertising elasticity for Product j
EDI is industry distribution elasticity for Product j.

This function captures marketing mix effects and allows nonlinearity in response to marketing variables. The nonlinearity is reflected in the parameters *EPI*, *EAI*, and *EDI*. For example, if $0 < EAI < 1$, the marginal sales response to advertising would be constantly decreasing as advertising increases. If $EAI = 0$, total group advertising does not affect the group's total sales. In general, *EAI* and *EDI* should be expected to fall between zero and plus one. The price parameter *EPI* should be negative because as price increases, sales should decrease. The parameters *EAI*, *EDI*, and *EPI* are elasticities and reflect the proportionate changes in the product group's sales resulting from a proportionate change in one variable.

Equation 1 reflects marketing mix effects since the sales response of one variable depends on other variables as established, for example, by differentiating

[3] For empirical consideration of decreasing returns to advertising, see [2, 3].

[4] This is similar in form to the Cobb-Douglas production function used by many authors in marketing, see [4]. For theoretical uses see [6, 7]. For empirical support see [8].

Equation 1 with respect to price. The marginal response to price changes (dX_{jI}/dP_{jI}) depends on the level of advertising and distribution. Differentiating Equation 1 with respect to the other variables will yield similar results.

Equation 1 is appropriate for this model since it satisfies the twin criteria of decision relevance and simplicity; that is, it includes nonlinear marketing mix effects in a simple form.[5] In addition, Equation 1 can be estimated by linear logarithmic regression.

Product Class Interdependencies

Two basic kinds of interdependencies exist, complementarity and substitutability. Substitutability implies an introspective consumer attitude of substituting one product for another under certain conditions; complementarity implies that one product will be purchased along with another product. Substitution or complementary effects of one product group with other product groups are essential to a product line model. To include nonlinearity and marketing mix effects in the consideration of interdependency, the general form of Equation 1 seems applicable. In considering intergroup effects the variables would relate to other groups, and a reasonable form would be:

$$(2)\quad b\, P_{IM}^{CPjM}\, A_{IM}^{CAjM}\, D_{IM}^{CDjM},$$

b is scale constant
P_{IM} is average price of product group M
A_{IM} is total advertising level for product group M
D_{IM} is total distribution level for product group M
$CPjM$ is cross price elasticity for Products j and M
$CAjM$ is cross advertising elasticity for Products j and M
$CDjM$ is cross distribution elasticity for Products j and M.

The equation's parameters—cross elasticities of price, advertising, and distribution—have theoretical economic content; they measure product interdependency. The cross price elasticity between Products 1 and 2 is:

$$CP12 = \frac{dx_1/x_1}{dP_2/P_2},$$

x_1 is sales of Product 1

P_2 is price of Product 2.

In general if the cross price elasticity is positive, the products are substitutes and if negative, the products are complements.[6] Since price is not the only appropriate variable for monitoring interdependencies, promotion and distribution cross elasticities should be considered. The cross advertising elasticity is:

$$CA12 = \frac{dx_1/x_1}{dA_2/A_2},$$

x_1 is sales of Product 1

A_2 is advertising for Product 2.

If this elasticity is positive, the goods demonstrate complementarity and if negative, substitutability. The same implications are true for distribution. Interdependencies should be monitored through several variables since a product may be a substitute with respect to one and a complement with respect to another.

Notation 2 is a good model choice since it allows nonlinear interdependency effects and considers the marketing mix effects between products as it retains a simple form for log-linear regressions.

The group marketing mix and intergroup product interdependencies can be combined to specify the total sales of one product class as:

$$(3)\quad X_{jI} = kP_{jI}^{EPI}A_{jI}^{EAI}D_{jI}^{EDI}(\Pi_M P_{IM}^{CPjM}A_{IM}^{CAjM}D_{IM}^{CDjM}),$$

where Π_M is product sum over M, $M \neq j$ and k is scale constant.

Intragroup Competitive Brand Effects

The market share a brand gets will reflect that brand's relative marketing effectiveness compared with that of other brands in the product group. Relative effectiveness was prespecified as relevant for the theoretical presumption that the consumer-buying process entails comparing the perceived utility of competing products. A simple form for representing relative market share effects would be by a firm's advertising expenditure compared with the total industry's advertising. However, this does not include the marketing mix effects of each brand. Since the consumer judges each product by its overall utility, brand choice could be formulated by representing each brand's mix effects and adding the relative effectiveness. A form for representing mix effects of a product was developed in Equation 1. A market share expression using this format and including relative mix effects is:

$$(4)\quad \text{Market share for Product } j \text{ in Firm 1} = \frac{P_{1j}^{SP1}A_{1j}^{SA1}D_{1j}^{SD1}}{\sum_i P_{ij}^{SPi}A_{ij}^{SAi}D_{ij}^{SDi}}$$

P_{ij} is price of Product j by Firm i

A_{ij} is advertising level for Product j by Firm i

D_{ij} is distribution level for Product j by Firm i

SPi is competitive price sensitivity for Firm i and Product j

[5] For a discussion of more complex response forms, see [11]. If Equation 1 is not empirically viable, more complex forms should be investigated.

[6] This reasoning is not valid for a product that violates the law of demand (e.g., a Giffin good) because as price increases sales increase.

SAi is competitive advertising sensitivity for Firm i and Product j

SDi is competitive distribution sensitivity for Firm i and Product j.

Equation 3 seems complicated, but it is the simplest equation that captures the prespecified relevant phenomena of marketing mix and the relative brand choice.[7] The parameters of the equation reflect the market share sensitivity of each brand's marketing variables.[8] These parameters are individualized to allow the consideration of product differentiation within each product group.

Given Equation 3, the effects of various strategies and counterstrategies can be related to the market share a firm will receive. For example, if Firm 1 is the price leader for a homogeneous product market ($EP1 = EPi$), price lowering by that firm would be followed by price lowering of other firms with no change in market share. However, industry effects described in the previous section may be produced. Given a strategy and set of counterstrategies, this expression could be used to consider intraproduct-group competitive brand effects.[9]

Demand, Cost and Profit Models for a Firm

The submodels developed in the previous three sections can be combined into one equation to describe the sales of one brand of a product class. The sales of Firm 1's brand in Product j class is:

$$(5) \quad x_j = kP_{jI}^{EPI}A_{jI}^{EAI}D_{jI}^{EDI}[\Pi_M P_{IM}^{CPjM}A_{IM}^{CAjM}D_{IM}^{CDjM}] \left[\frac{P_{1j}^{SP1}A_{1j}^{SA1}D_{1j}^{SD1}}{\sum_i P_{ij}^{SPi}A_{ij}^{SAi}D_{ij}^{SDi}}\right],$$

where Π_M is product sum over M, $M \neq j$, and other notations as previously defined. Given constant direct and cross elasticities and sensitivities, this equation represents the demand for one product of a firm's product line. This formulation could be extended to include more than three marketing variables by specifying the appropriate direct and cross elasticities and sensitivities.

A firm's total revenue is the sum of each product's price times its sales. To calculate profit, costs must be specified. The costs may be in the simple form of fixed plus variable costs, but if the products share common production resources, this is unlikely. If there are production interdependencies, a linear programming model designed to minimize the cost of producing specified quantities of the products could be used. Successive runs of this model or cost records could provide the data for estimating an interdependent cost function such as:

$$(6) \quad TVC_j = AVC_j\ (x_j)\ \Pi_M\ (x_M)^{CCjM},$$

TVC_j is total variable cost of producing the firm's brand of Product j

AVC_j is average variable cost function for the firm's brand of Product j, if produced independently of other products

x_j is quantity of brand of Product j produced

x_M is quantity of brand of Product M produced, ($M \neq j$)

CCjM is cross cost elasticity of firm's brands Products j and M, ($M \neq j$).

Subtracting the variable cost and fixed production, advertising and distribution costs from the total revenue will yield total profit.

Combining the cost and demand equations in the calculation of profit results in a simple model that includes the phenomena that were considered a priori to have high decision relevance.

Output of Model

Assuming the firm's problem in the short run is to maximize the total profit subject to existing technical, managerial, financial, and production constraints, the output of the model should be the best marketing strategy for each brand in the firm's product line. This requires the optimization of the model's profit function which is difficult since the model is not amenable to mathematical programming or other analytical techniques. However, it may be solved by an iterative search routine.[10]

The feasibility of gaining the described output from the model rests on the ability to generate meaningful input and on the presence of a practical solution method. The direct and cross elasticities could be estimated on a subjective basis that reflects the decision maker's best judgment. This approach might be justified since the decision must be made and if the model is not used, a simpler and perhaps less accurate decision procedure would be used. However, subjective inputs should be used only after all empirical information relating to the problem has been considered.

The model developed could be especially useful to firms using brand managers since it can be a basis of allocating resources to each brand in the product line. The brand manager concept artificially imposes independence between specific products in the line by delegating products to competing brand managers. But if resources are allocated on the basis of product interdependencies at the top marketing management level, the motivational advantages of the brand manager con-

[7] For a discussion of a more complex form including advertising interdependency in the consideration of competitive effects, see [10].

[8] The sensitivities appear to be similar to elasticities, but they are not elasticities. They do not represent proportionate changes in market share as the result of proportionate changes in the variable. However, they do represent the sensitivity of the market share to changes in the marketing mix for each firm. Equation 4 is similar to Kotler (see [6]) except the sensitivities are subscripted to allow the possibility of differentiated products and response.

[9] For game theoretic considerations of this form, see [6, 10].

[10] See [14] and the application section of this article.

Table 1
PRODUCT GROUP ELASTICITIES AND CROSS ELASTICITIES SEPARATED BY PRODUCER AND COMPETITORS[a]

Elasticity	*Class*	*Product 1* ($j = 1$)	*Product 2* ($j = 2$)	*Product 3* ($j = 3$)
Price elasticity of Product j (*EPI*)	us	−3.83[b]	−1.86[b]	−2.64[b]
	them	—[e]	−1.01[d]	.26
Facing elasticity of Product j (*EFI*)	us	−.173	.113	.157
	them		.837[b]	.038
Cross price elasticity between Products j & 1 (*CPj*1)	us		.266	1.28
Cross facing elasticity between Products j & 1 (*CFj*1)	us		−.081	.162
Cross price elasticity between Products j & 2 (*CPj*2)	us	−.91		1.67[c]
	them	−.55		−.61
Cross facing elasticity between Products j & 2 (*CFj*2)	us	.24		−.105
	them	.38[d]		.41[d]
Cross price elasticity between Products j & 3 (*CPj*3)	us	−.29	−1.55[b]	
	them	.207[c]	.136[b]	
Cross facing elasticity between Products j & 3 (*CFj*3)	us	.11	−.42[d]	
	them	−.301[d]	.09	
R^2		.52[b]	.61[b]	.51[b]

[a] Significance is based on one tail test for direct elasticities and two tail tests on cross elasticities.
[b] Significant at .01 level.
[c] Significant at .05 level.
[d] Significant at .10 level.
[e] There was no competition in Product 1's market.

NOTE: "us" is our brand in product group j; "them" is all other brands in product group j
$CPiN > 0 \Rightarrow$ substitutes $CFiN > 0 \Rightarrow$ complements
$CPiN < 0 \Rightarrow$ complements $CFiN < 0 \Rightarrow$ substitutes.

cept and the use of product line resources to maximize total line profits can be compatible.

AN EMPIRICAL APPLICATION

To test the descriptive adequacy and usefulness of the proposed model in a real product line problem, 100 grocery store audits of a three product line of related, frequently purchased consumer goods were used.[11] These product line data were used to estimate the parameters of the product line model, and an on-line computer search program derived the optimum marketing mix for each product in the producer's line.

The audited product line contained three classes of products that served the same food need but had different product features. Product group 1 was a new product and only the firm to be examined offered a brand in this class. The competitors in product groups 2 and 3 were aggregated into one competitor in each market. The aggregation resulted in a firm with a three product line and brands which faced no competitors in product group 1, one competitor in product group 2, and one competitor in product group 3.

In each product class the brand, shelf price, number of facings, deals, and special displays were recorded in the audits. Over ninety-five percent of the data were recorded with cents-off or bonus-size deals. Hence, dealing was not considered a separate variable since it could be reflected in the price per unit. Special displays occurred so infrequently (less than one percent of the data) that they were not considered in the analysis. The audits did not monitor national or local advertising in the test area. It was assumed that none of the brands received a disproportionate amount of local advertising at any of the audited stores, and advertising was not considered as a variable in the testing.

Model Parameter Estimation

Product Class Marketing Effects and Product Interdependency. The product group elasticities and cross elasticities for each product class were obtained by linear logarithm regressions of Equation 3 where distribution is represented by the number of total facings of the product group and where advertising is omitted. All the direct price elasticities obtained from this regression were negative, and all facings' elasticities were positive as expected and significant at least at the .05 level. The significant cross elasticities of price and facings for the products indicated that the three product groups were basically complementary. This complementarity did not agree with a priori feelings and past studies that indicated these kinds of products could be competing for the same buyers in the general product class.[12] This finding implies that the prespecification of the model's product interdependency section was not satisfactory.

To explore alternative forms for updating the model, the interdependencies between our brand in a product group and other product groups were postulated to be different from other brands in the group. To evaluate this updated model structure, the industry price and facing data for each product group were subdivided into "us" and "them" classes. Us was our brand price and facings and them was the average price and facing for all other competitors. The elasticities and cross elasticities for the firm us and the competitors them in each product are indicated in Table 1.

An examination of cross elasticities for Product 1 indicates that Product 1 was complementary to both our brand and other competitive brands of Product 2. Cross elasticities for Product 3 indicate that Product 1 was complementary to our brand of Product 3 but shows substitution effects with other brands in Product 3.

[11] The author thanks Samuel G. Barton and I. J. Abrams of the Market Research Corporation of America for use of these data. The product line is not identified to protect the interests of the producer and MRCA.

[12] See [5].

Product group 2 was complementary to our brand in the Product 3 group but was a substitute for competitive brands in Product 3's market. Although the initial regression showed Products 1 and 2 to be complementary to Product 3, the new regressions (Table 1) indicate that complementarity was with our brands of Product 3, and that Products 1 and 2 were competitive with other brands of Product 3. This is the interdependency pattern a marketing manager would desire.

Product group 3 showed no significant interdependencies with Product 1 but displayed some interesting relationships with product group 2. $CP32_{us} = 1.67$ indicates our brand of Product 2 was a substitute for Product 3. But $CP23_{us} = -1.55$ indicates Product 2's sales were complementary to our brand of Product 3. The asymmetry in the interdependency is significant at the five percent level and the elasticities are large, i.e., greater than one.[13]

That product group 3 felt substitution effects from our brand of Product 2 is understandable. As the price of our brand of Product 2 increased, our brand buyers substituted Product 3. That product group 2 felt complementary effects from our brand of Product 3 is more difficult to explain. $CP23_{US} = -1.55$ so that a 10 percent reduction in the price of our brand of Product 3 caused a 15.5 percent increase in the sales of Product 2. This could have occurred if buyers of our brand of Product 3 had also bought Product 2 when they perceived low prices for this class of goods. If the perception is based on the price of Product 3, lowering our brand's prices of Product 3 could have caused a perception of low prices for these buyers. They might have bought more of Product 2 with little sensitivity to its price. The income effect caused by lower prices in Brand 3 also indicates complementarity. This increase in real income might have led to additional purchases of Product 2.

Except for the asymmetry, the new regressions indicate that the products within the firm's product line are complementary but are substitutes for products in other firms' product lines. The split product group regressions explained 30 percent more of the variance in the data than the earlier regressions so updating the model for split interactions is advisable.

Intragroup Competitive Brand Sensitivities. The estimation of the competitive sensitivities to be used in describing market share effects (see Equation 4 with facings representing D and omitting A) was carried out by a computer program to minimize the total variation between the actual market shares and the market shares predicted by Equation 4 given a set of sensitivities, observed prices, and observed facings. The estimation was executed on the MIT computation center compatible time-sharing system. The interactive ability of an on-line system was used in a conversational program that asks the researcher or manager to supply initial estimates of the price and facing sensitivities for the firm and its competitors. These initial sensitivities are incremented by an amount prescribed by the manager. The number of incremental steps to be taken for each sensitivity is also an on-line input supplied by the manager. All combinations of the initial and incremented sensitivities are evaluated; the set of sensitivities producing the minimum total variation for the audit data points is recorded.

Then the manager is asked to supply a new increment and number of steps for the search. The next evaluation uses the best past estimates as initial values. By continuing this process the manager can guide the search until it has reached a prescribed level of accuracy. This procedure does not guarantee that the optimal fit has been achieved; rather, it is a heuristic procedure based on the manager's best judgment and the computational power of a high speed computer.[14]

The minimum variation estimates for the two competitive product markets are shown in the tabulation. The estimates explain 24 percent of the variance of the market shares in Product 2 and 54 percent of Product 3's market shares. This empirical success adds confidence to the a priori specification of the competitive phenomena.[15]

Producer	*Product 2*		*Product 3*	
	Price	*Facing*	*Price*	*Facing*
Our firm	−.27	1.31	−.855	1.13
Competitor	.00	.75	−1.24	1.20

The competitive sensitivities in Product 2 indicate that the competitor has little or no effect on market share by his change in prices while changes in our price have a negative sensitivity. The facings sensitivities of Product 2 indicated our firm's facings were 1.5 times as effective as the competitor's in producing market share changes. The Product 3 competitive estimates indicated our firm's price and facing variables were less effective in changing market share than the competitor's.

The estimation of the competitive sensitivities completes the parameter estimation for the model's demand equation (see Equation 5). The remaining demand input is the strategy competitors were expected to use;

[13] This asymmetry was difficult to accept, so stepwise multiple regressions were run for our brand sales in markets for products 2 and 3. The asymmetry again appeared at the five percent significance level. Analysis of the correlation matrix showed little multicolinearity between the variables. Since the data was taken at one point in time, autocorrelation in the data was not suspected. If there had been multicolinearity in the autocorrelation, this might have caused the asymmetry. The elasticity of store size averaged .08 for the relevant brands. This appeared reasonable and supported the assumption that the stores represented a simple sample with respect to market responses to the variables.

[14] For details of the search technique, see [13].

[15] The prespecified structure was also reinforced since simple log-linear regressions of the two firms' prices and facings against sales produced low R^2 values and unreasonable coefficients in the two products. This suggests that the proposed form is a relevant level of detail.

Table 2
SEARCH RESULTS

Variable	*Reference*	*No interaction*	*Product group interaction*	*Split product group interaction*
Price of Product 1	20.5	16.8	16.8	16.8
" " " 2	17.0	22.0[a]	22.0[a]	22.0[a]
" " " 3	21.0	26.0[a]	26.0	16.0[a]
Facings of Product 1	2.00	.75[a]	.75[a]	.75[a]
" " " 2	4.00	3.50	6.25	6.50
" " " 3	2.00	3.75	1.00	.75[a]
Total Profit	382,600	594,500	585,180	908,700
Profit for Product 1	107,100	128,000	158,700	154,000
" " " 2	184,600	299,800	324,400	704,800
" " " 3	90,900	166,700	102,000	49,900
Sales for Product 1	1,500,000	3,083,200	3,722,200	3,625,600
" " " 2	2,500,000	2,421,700	2,611,100	5,537,100
" " " 3	1,000,000	1,167,400	762,200	1,165,200
Market share for Product 1	100%	100%	100%	100%
" " " " 2	29.64%	28.76%	31.01%	31.1%
" " " " 3	74.79%	70.03%	72.03%	80.37%

[a] These values are at the upper or lower limit of the data used for estimation.

they were expected to be nonadaptive with respect to the number facings and price for Product 2 and be followers with respect to Product 3. The final input to the model is the cost function (see Equation 6). In this test application the costs for each product were independent, and the marginal costs were considered constant over the meaningful range of production.

Optimization of Product Line Profit

The maximization of the model's product line profit was carried out by an on-line computer search routine.[16] The trial and error routine began by evaluating a reference set of prices and facings for each of the three products and then examining a range of values on each side of the reference values in discrete steps. The range and steps were specified from the remote computer console. Given the increments and ranges all combinations of the trial values were run, and the best total product line profit based on Equations 1 to 6 with the estimated parameters was recorded. After the first series of trials have been reported, the manager can respecify the step intervals and ranges. By continuing this process he can achieve the desired level of accuracy. One constraint was placed on the search—the total facings had to be less than eight for our product line. This constraint forces the search routine to allocate the shelf space between the products in the line.

The search program results are shown in Table 2. The first column gives the price per unit, facings weighted by package size, profit, sales, and market share for our brand of each product at the reference level. The reference results are based on the average price and facings for each group as observed in the audits.

The next question was whether the interdependencies between products should be considered at the aggregate product group level or at the split-product group level. The empirical estimation indicated that the model should be updated for the split, but if the decision were insensitive to the added complexity of the split, it might be omitted. A sensitivity analysis determined if the optimum marketing strategy was different under varying interaction assumptions.

To establish a reference base in evaluating decision sensitivity, the most profitable program with no interactions was found. It was considerably different from the reference program. The price of Product 1 was decreased but the prices for Products 2 and 3 were increased. The facing's allocation was also altered. Products 1 and 2 received fewer facings but Product 3 received more. The result of an improvement in profit of over 50 percent and an increase in the sales of each product implies that on the basis of the model the existing strategy was nonoptimal.

The best program with aggregate product group interaction led to the same price structure as the no interaction case. The facing allocation was changed, however (see Table 2). The facings were concentrated in Product 2 because of the complementarity between the facings of Product 2 and sales of Products 1 and 3 in the regressions.

[16] See [13].

The next phase of the sensitivity analysis was to determine the maximum profit for the most empirically valid case—split-product-group interactions. In this case a different pricing strategy should be used. The price of Product 3 is lower than even the reference price primarily because of the complementarity of the price of our brand of Product 3 and the sales of Product 2 (see Table 1). The product line profits in this analysis were 50 percent greater than in the no interaction or group interaction cases. Additional profits occurred in Product 2, but the profits in Product 3 decreased almost 70 percent because of the asymmetric product interdependencies between our brands of Products 2 and 3. The decision output is sensitive to the splitting of the interactions; since this is the most empirically viable model structure, the model should be updated to reflect the differences in the product interdependencies between our brands and other brands in other product groups.

The output of the optimization and sensitivity testing can be summarized in the recommendations that the price of Product 1 be lowered, the price of Product 2 be raised, and the facings allocation be more concentrated on Product 2. The price of Product 3 should be lowered if the asymmetric interdependency between Products 2 and 3 is real as it appears to be. Additional study is necessary to ascertain the underlying behavior that generates the asymmetric condition, but updating the model for split interactions is recommended since application of the model identified significant product interdependencies and recommended changes in the marketing mix of the products in the line so that the interdependencies could be exploited for additional profit. The updated model possesses reasonable descriptive adequacy and decision relevance.

SUMMARY AND EXTENSIONS

This article presented an a priori product line model for finding the best marketing mix for each product in a line. The model includes aggregate product group marketing mix, product interdependency, and competitive brand effects. The initial testing of the model suggests that the basic structure is appropriate and the model deserves additional consideration, testing, and development.

The model analysis could be extended in several ways. First, the model was a static one period model; the analysis could be extended to include carry-over effects and the problem of multiperiod marketing mix determination. Second, the model test application examined only three related products; it would be useful to expand the test product definitions to include other classes of products or to narrow the product definition to consider package sizes in each product. A hierarchy of interdependencies exists, and a sequential application of the model to increasingly more specific product definitions would be appropriate. Third, the multiple regression and iterative search routines used to estimate the model's parameters were applied to a limited data base of 100 store audits. Although reasonable descriptive adequacy was found, it would be useful to have an information system that builds a data bank on the products' performance to obtain more accurate input estimates and to test more complex response forms. And consideration of the effects of adding or dropping a product from the line and the effects of this action on the marketing mix would extend the analysis.[17]

REFERENCES

1. Arnold E. Amstutz, *Computer Simulation of Competitive Market Response,* Cambridge, Mass.: MIT Press, 1967.
2. B. Benjamin and J. Maitland, "Operational Research and Advertising: Some Experiments in the Use of Analogies," *Operational Research Quarterly,* 9 (September 1958), 207–17.
3. B. Benjamin, W. P. Jolly, and J. Maitland, "Operational Research and Advertising: Theories of Response," *Operational Research Quarterly,* 11 (December 1960), 205–18.
4. Paul H. Douglas, *Theory of Wages,* New York: The Macmillan Company, 1934.
5. Peter L. Henderson, James F. Hind, and Sidney E. Brown, "Sales Effects of Two Campaign Themes," *Journal of Advertising Research,* 1 (December 1961), 2–11.
6. Philip Kotler, "Competitive Strategies for New Product Marketing Over the Life Cycle," *Management Science,* 12 (December 1965), 104–19.
7. Alfred A. Kuehn and Doyle L. Weiss, "Market Analysis Training Exercise," *Behavioral Science,* 10 (January 1965), 51–62.
8. William F. Massy and Ronald E. Frank, "Short Term Price and Dealing Effects in Selected Market Segments," *Journal of Marketing Research,* 3 (May 1965), 171–85.
9. Francesco M. Nicosia, *Consumer Decision Processes,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1966.
10. Melvin Shakun, "A Dynamic Model for Competitive Marketing in Coupled Markets," *Management Science,* 12 (August 1966), 525–30.
11. Glen L. Urban, "Sprinter: A Tool for New Product Decision Makers," *Industrial Management Review,* 8 (Spring 1967), 43–55.
12. ———, "A New Product Analysis and Decision Model," *Management Science,* 14 (April 1968), 490–517.
13. ———, "An On-Line Technique for Estimating and Analyzing Complex Models," in Reed Moyer, ed., *Changing Marketing Systems,* Winter Conference Proceedings, American Marketing Association, 1967, 322–7.
14. Douglass Wilde and Charles S. Beightler, *Foundations of Optimization,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967.

[17] See [12].